

Stationary points

Difficulty : Medium

Question Paper 1

Level	AS & A Level
Subject	Maths - Pure
Exam Board	Edexcel
Topic	Differentiation
Sub-Topic	Stationary points
Difficulty	Medium
Booklet	Question Paper 1

Time allowed: 74 minutes

Score: /62

Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E	U
>76%	61%	52%	42%	33%	23%	<23%

Question 1

A diesel lorry is driven from Birmingham to Bury at a steady speed of v kilometres per hour. The total cost of the journey, £ C , is given by

$$C = \frac{1400}{v} + \frac{2v}{7}.$$

(a) Find the value of v for which C is a minimum. (5)

(b) Find $\frac{d^2C}{dv^2}$ and hence verify that C is a minimum for this value of v . (2)

(c) Calculate the minimum total cost of the journey. (2)

(Total 9 marks)

Question 2

A lorry is driven between London and Newcastle.

In a simple model, the cost of the journey £ C when the lorry is driven at a steady speed of v kilometres per hour is

$$C = \frac{1500}{v} + \frac{2v}{11} + 60$$

(a) Find, according to this model,

(i) the value of v that minimises the cost of the journey,

(i) the minimum cost of the journey.

(Solutions based entirely on graphical or numerical methods are not acceptable.) (6)

(b) Prove by using $\frac{d^2C}{dv^2}$ that the cost is minimised at the speed found in (a)(i). (2)

(c) State one limitation of this model. (1)

(Total 9 marks)

Question 3

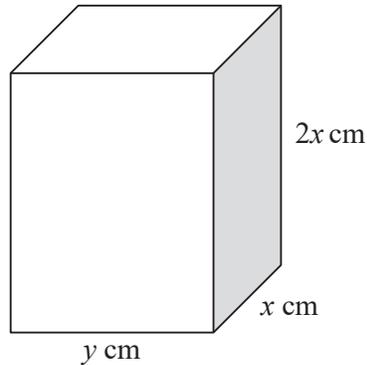


Figure 4

Figure 4 shows a solid brick in the shape of a cuboid measuring $2x$ cm by x cm by y cm.

The total surface area of the brick is 600 cm^2 .

(a) Show that the volume, $V \text{ cm}^3$, of the brick is given by

$$V = 200x - \frac{4x^3}{3}. \quad (4)$$

Given that x can vary,

(b) use calculus to find the maximum value of V , giving your answer to the nearest cm^3 . (5)

(c) Justify that the value of V you have found is a maximum. (2)

(Total 11 marks)

Question 4

Figure 4

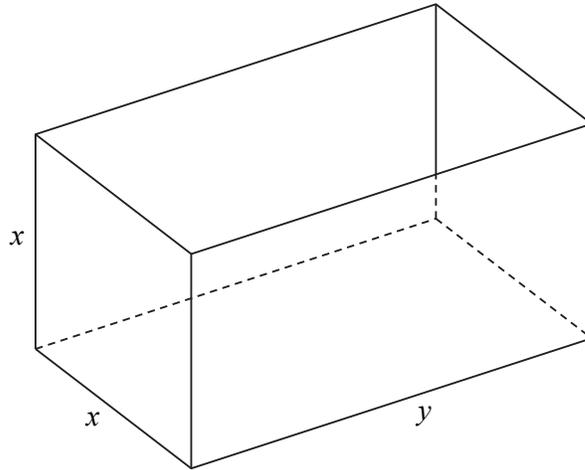


Figure 4 shows an open-topped water tank, in the shape of a cuboid, which is made of sheet metal. The base of the tank is a rectangle x metres by y metres. The height of the tank is x metres.

The capacity of the tank is 100 m^3 .

(a) Show that the area $A \text{ m}^2$ of the sheet metal used to make the tank is given by

$$A = \frac{300}{x} + 2x^2. \quad (4)$$

(b) Use calculus to find the value of x for which A is stationary. (4)

(c) Prove that this value of x gives a minimum value of A . (2)

(d) Calculate the minimum area of sheet metal needed to make the tank. (2)

(Total 12 marks)

Question 5

The volume V cm³ of a box, of height x cm, is given by

$$V = 4x(5 - x)^2, \quad 0 < x < 5$$

(a) Find $\frac{dV}{dx}$. **(4)**

(b) Hence find the maximum volume of the box. **(4)**

(c) Use calculus to justify that the volume that you found in part (b) is a maximum. **(2)**

(Total 10 marks)

Question 6

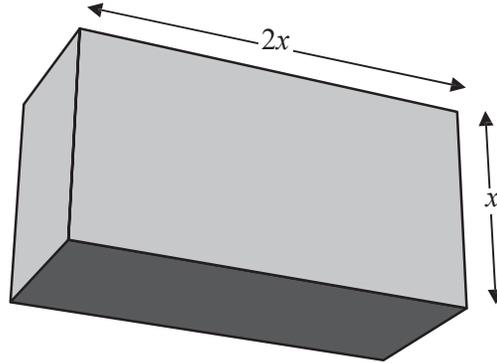


Figure 2

A cuboid has a rectangular cross-section where the length of the rectangle is equal to twice its width, x cm, as shown in Figure 2.

The volume of the cuboid is 81 cubic centimetres.

(a) Show that the total length, L cm, of the twelve edges of the cuboid is given by

$$L = 12x + \frac{162}{x^2} \quad (3)$$

(b) Use calculus to find the minimum value of L . (6)

(c) Justify, by further differentiation, that the value of L that you have found is a minimum. (2)

(Total 11 marks)