Finding Areas
Difficulty: Medium

Question Paper 2

Level	A Level only
Subject	Maths - Pure
Exam Board	Edexcel
Topic	Integration
Sub-Topic	Finding Areas
Difficulty	Medium
Booklet	Question Paper 2

Time allowed: **48 minutes**

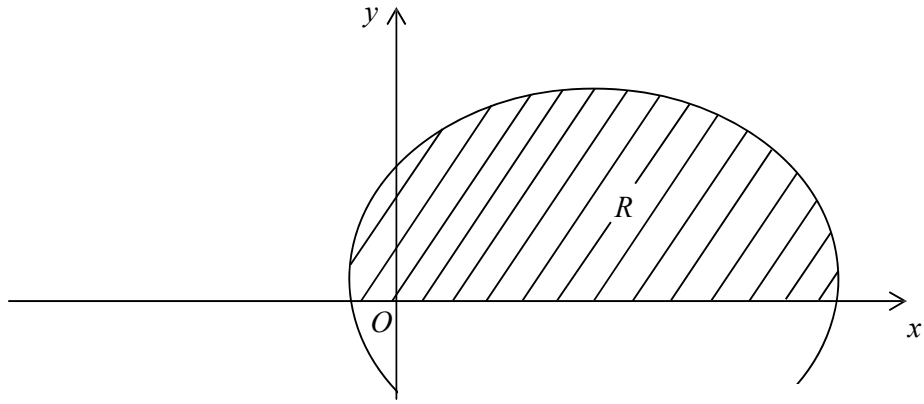
Score: **/40**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>76%	61%	52%	42%	33%	23%	<23%

Figure 2



The curve shown in Figure 2 has parametric equations

$$x = t - 2\sin t, \qquad y = 1 - 2\cos t, \qquad 0 \le t \le 2\pi.$$

(*a*) Show that the curve crosses the *x*-axis where $t = \frac{\pi}{3}$ and $t = \frac{5\pi}{3}$. **(2)**

The finite region *R* is enclosed by the curve and the *x*-axis, as shown shaded in Figure 2.

(*b*) Show that the area *R* is given by the integral

$$\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2\cos t)^2 \, \mathrm{d}t.$$

(3)

(*c*) Use this integral to find the exact value of the shaded area.

(7)

(Total 12 marks)

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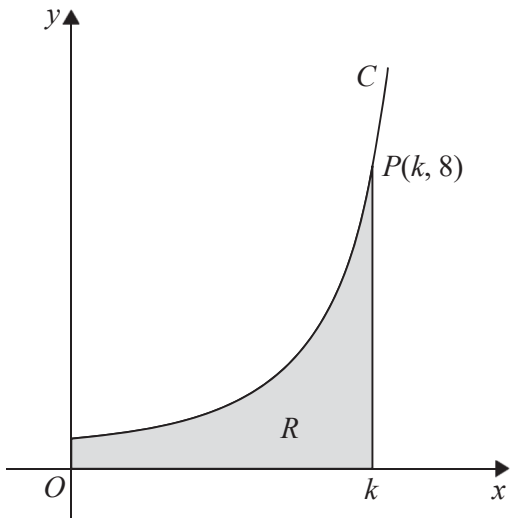

Diagram not drawn to scale

Figure 4

Figure 4 shows a sketch of part of the curve C with parametric equations

$$x = 3\theta\sin\theta, \qquad y = \sec^3\theta, \qquad 0 \leqslant \theta < \frac{\pi}{2}$$

The point $P(k, 8)$ lies on C, where k is a constant.

(a) Find the exact value of k. **(2)**

The finite region R, shown shaded in Figure 4, is bounded by the curve C, the y-axis, the x-axis and the line with equation $x = k$.

(b) Show that the area of R can be expressed in the form

$$\lambda \int_{\alpha}^{\beta} \left(\theta \sec^2 \theta + \tan \theta \sec^2 \theta \right) \mathrm{d}\theta$$

where λ, α and β are constants to be determined.

(4)

(c) Hence use integration to find the exact value of the area of R.

(6)

(Total 12 marks)

Figure 3 shows part of the curve with equation $y = x\sin^2 x$. The finite region bounded by the line with equation $x = \frac{\pi}{2}$, the curve and the x-axis is shown shaded in the diagram.

Find the area of the shaded region.

(7 marks)

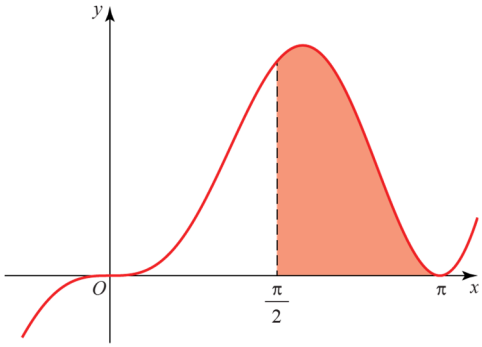

Figure 3

(Total 7 marks)

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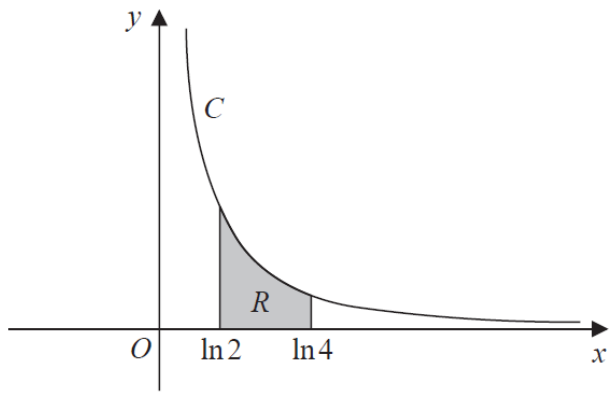

Figure 4

Figure 4 shows a sketch of the curve C with parametric equations

$$x = \ln(t + 2), \quad y = \frac{1}{t+1}, \qquad t > -\frac{2}{3}$$

(a) State the domain of values of x for the curve C.

(1)

The finite region R, shown shaded in Figure 4, is bounded by the curve C, the line with equation $x = \ln 2$, the x-axis and the line with equation $x = \ln 4$

(b) Use calculus to show that the area of R is $\ln\left(\frac{3}{2}\right)$.

(8)

(Total 9 marks)