

Mathematics: applications and interpretation
Standard level
Paper 2

Save My Exams Practice Paper

1 hour 30 minutes

Instructions to candidates

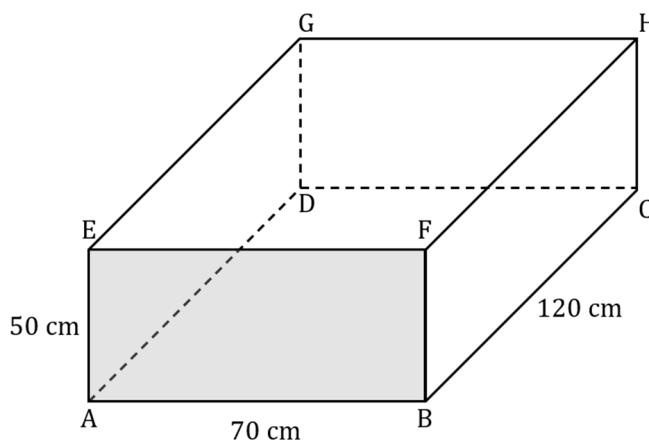
- A graphic display calculator is required for this paper.
- Answer all questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 15]

The Amazing Box Company manufactures rectangular gift boxes.

Their largest box size has length 120 cm, width 70 cm and height 50 cm as shown in the diagram.



Customers often ask for other measurements relating to the gift boxes.

(a) Calculate the length GB. [2]

(b) Calculate the surface area of the box in m^2 . [2]

Each month, The Amazing Box Company expects to sell x hundred gift boxes.

It is known that $\frac{dP}{dx} = -2x + 480$, $x \geq 0$, where P is the monthly profit, in euros, from the sale of x hundred boxes.

(c) Find the number of boxes that should be sold each month to maximize the profit. [3]

It is also known that the company makes a profit of €960 in a month where it sells 15 000 boxes.

(d) Find $P(x)$. [5]

(e) Find the least number of boxes which must be sold each month in order to make a profit. [3]

2. [Maximum mark: 17]

A catapult is designed to project a boulder from ground level. The distance travelled horizontally by the boulder, x m, and the height of the boulder above ground level, h m were recorded. The results are summarized in the following table.

Horizontal distance (x m)	0	20	60
Vertical distance (h m)	0	16	24

This information was used to create Model A, where h is a function of x , $x \geq 0$.

$$\text{Model A: } h(x) = px + qx^2 \text{ where } p, q \in \mathbb{Z}$$

At a horizontal distance 20 m, Model A can be represented by the equation $p + 20q = 0.8$.

- (a) (i) Write down a second equation to represent Model A, when the horizontal distance is 60 m.
[4]
- (ii) Find the values of p and q .
[4]
- (b) Find the coordinates of the vertex on the graph of $y = h(x)$ and describe what this point represents in the context of the question.
[2]
- (c) Sketch the graph of $y = h(x)$ for $0 \leq x \leq 120$ and $-30 \leq y \leq 30$, clearly showing the vertex.
[3]
- (d) Hence identify why Model A may not be appropriate once the boulder has covered a horizontal distance of 100 m.
[1]

Amongst other findings, it was found that the catapult was releasing boulders from a height 2 m above ground level. In light of these findings, a revised model, Model B, for the position of the boulder was created.

$$\text{Model B: } h(x) = 0.84x - 0.012x^2 + 2$$

- (e) Use Model B to calculate an estimate for the height of the boulder at the point when it has covered 22 m horizontally.
[2]

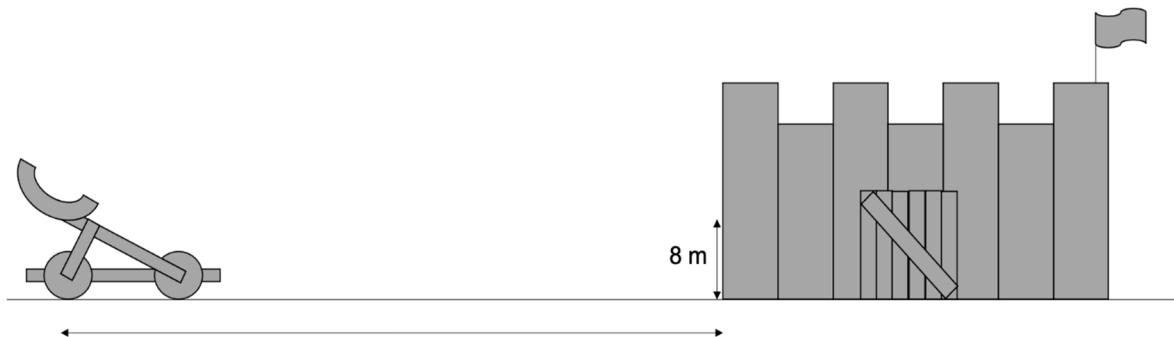
The actual height of a boulder that has covered 22 m horizontally is 14.7 m.

- (f) Calculate the percentage error in the estimate in part (e).
[2]

(This question continues on the following page)

(Question 2 continued)

The catapult is used to recreate a famous Roman battle. In order to cause as much damage to the castle as possible with a single shot, the boulder should hit the castle at a point no lower than 8 m from ground level.



- (g) Using Model B find the maximum distance from the castle that the catapult should be located.

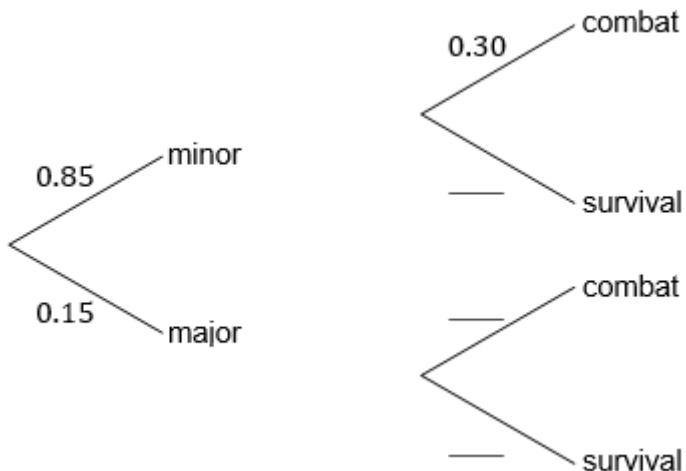
[3]

3. [Maximum mark: 14]

In an online game there are various ‘power-ups’ that players can discover while progressing through the game. The power-ups come in two sizes, major and minor, and they all belong to one of two categories, either combat or survival.

It is known that 85 % of the power-ups in the game are minor power-ups while the other 15 % are major power-ups. Of the major power-ups it is known that 60 % are combat power-ups, while of the minor power-ups it is known that 30 % are combat power-ups.

- (a) Using the given information, **copy** and complete the following tree diagram. [2]



- (b) Power-ups that are discovered in the game occur at random.

- (i) Find the probability that when a power-up is discovered it will be a survival power-up.
(ii) Given that a power-up is a combat power-up, find the probability that it is a major power-up. [4]

(This question continues on the following page)

(Question 3 continued)

In the online game it is possible to purchase ‘mystery bundles’ containing random assortments of power-ups. According to the game’s website, the power-ups in each mystery bundle should be distributed as follows.

Type	Sword	Lasgun	Armour	Rations	Medical	Other Gear
Percentage (%)	15	10	15	30	10	20

Dylan and his teammates have joined together to purchase a ‘mega mystery bundle’ containing 60 power-ups. After going through the bundle, they find it contains the following frequencies of each type of power-up:

Type	Sword	Lasgun	Armour	Rations	Medical	Other Gear
Observed frequency	4	2	5	21	9	19

Dylan and his friends are upset that they did not receive more weapons and armour. Some of them think that they were simply unlucky this time around, while others insist that the website’s claims about the contents of the bundles are misleading.

Dylan is also studying Mathematics as part of his IB course, so he decides to investigate if the sample is consistent with the website’s claims by conducting a χ^2 goodness of fit test. The test is carried out at a 5 % significance level.

(c) Write down the null hypothesis for this test. [1]

(d) **Copy** and complete the following table in your answer booklet. [2]

Type	Sword	Lasgun	Armour	Rations	Medical	Other Gear
Expected frequency						

(e) Write down the number of degrees of freedom. [1]

(f) Find the p -value for the test. [2]

(g) State the conclusion of the test. Give a reason for your answer. [2]

4. [Maximum mark: 17]

In this question, give all answers to two decimal places.

Natasha needs to purchase new sound equipment for her band, the Oscillots. The sound equipment costs £12000, and as she does not have enough money to pay the full amount up front, she is considering two different methods of financing the purchase.

Option 1:

Natasha is eligible to receive a personal loan from her bank for the full amount of £12000. It will be a 5 year loan at a nominal annual interest rate of 3.9 % **compounded monthly**. Repayments are made each month.

- (a) (i) Find the repayment made each month.
- (ii) Find the total amount paid for the sound equipment.
- (iii) Find the interest paid on the loan.

[7]

Option 2:

The music shop from which Natasha is buying the sound equipment offers its own financing scheme. This scheme offers her a 5 year loan at a nominal annual interest rate of r % **compounded quarterly**. The terms of the loan require a 10 % payment up front and quarterly repayments of £600.

- (b) (i) Find the amount to be borrowed for this option.
- (ii) Find the annual interest rate, r .
- (c) State which option Natasha should choose. Justify your answer.

[5]

[2]

The sound equipment Natasha is buying depreciates at an annual rate of 15 % per year.

- (d) Find the value of the sound equipment five years after it is purchased.

[3]

5. [Maximum mark: 17]

In a dressage competition judges award scores for each test attempted by competitors. For the “rhythm test” 10 competitors were awarded scores on a scale from 0 to 10 (10 being the highest) by a US and a UK judge. The scores are collated in the table below.

Competitor	A	B	C	D	E	F	G	H	I	J
US judge (x)	7.5	9.5	6.5	8	6.5	7	5.5	9	4	7
UK judge (y)	7.5	9	7	8.5	6	7.5	5	8	5	7.5

- (a) (i) Write down the value of the Pearson’s product–moment correlation coefficient, r .
 (ii) Using the value of r , interpret the relationship between the US judge’s scores and the UK judge’s scores. [4]
- (b) Write down the equation of the regression line y on x . [2]
- (c) (i) Use your regression equation from part (b) to estimate the US judge’s score when the UK judge awards a score of 5. Give your answer to the nearest half mark.
 (ii) Briefly explain whether or not the regression equation from part (b) predicts both judges awarding a score of 0 to the same competitor. [4]

Following complaints from competitors about the judges’ scores, officials for the competition ran an investigation that, amongst other things, involved finding the Spearman’s rank correlation coefficient.

- (d) **Copy** and complete the information in the following table. [2]

Competitor	A	B	C	D	E	F	G	H	I	J
US judge rank		1				5.5				5.5
UK judge rank		1					9.5		9.5	

- (e) (i) Briefly explain why there is no rank of 10 in the UK judge’s rankings.
 (ii) Find the value of the Spearman’s rank correlation coefficient, r_s . [4]

Following their investigation, officials decreased the UK judge’s score for competitor B from 9 to 8.5.

- (f) Explain, giving a reason, whether or not you would expect the value of the Spearman’s rank correlation coefficient, r_s , to change. [1]