UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
International General Certificate of Secondary Education

CANDIDATE NAME									
CENTRE NUMBER						CANDIDATE NUMBER			

7157806085

MATHEMATICS **0580/41**

Paper 4 (Extended) **October/November 2010**

2 hours 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator
Mathematical tables (optional)
Geometrical instruments
Tracing paper (optional)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** questions.
If working is needed for any question it must be shown below that question.
Electronic calculators should be used.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π use either your calculator value or 3.142.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 130.

This document consists of **16** printed pages.

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1 **(a)** In 2008 the total number of tickets sold for an athletics meeting was 3136.
The ratio child tickets sold : adult tickets sold = 17 : 32.

(i) How many child tickets were sold?

Answer(a)(i) ..... [2]

(ii) Child tickets cost \$2 each and adult tickets cost \$4.50 each.

Show that the total amount received from the sale of the tickets in 2008 was \$11 392.

Answer(a)(ii)

[2]

(b) In 2009 the amount received from the sale of tickets for the athletics meeting was \$12 748.

Calculate the percentage increase in the amount received from 2008 to 2009.

Answer(b) ..... % [3]

(c) In 2008 the amount of \$11 392 was 28% more than the amount received in 2007.

Calculate how much was received in 2007.

Answer(c) \$ ..... [3]

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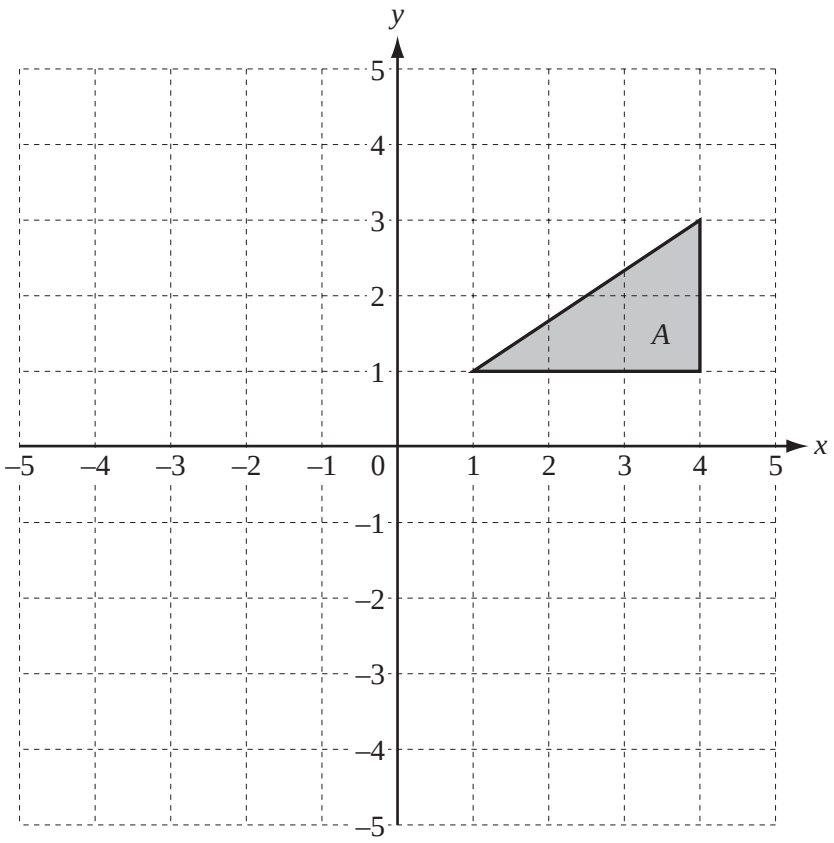
2 (a)



(i) Draw the image when triangle A is reflected in the line $y = 0$.
Label the image B. [2]

(ii) Draw the image when triangle A is rotated through 90° anticlockwise about the origin.
Label the image C. [2]

(iii) Describe fully the **single** transformation which maps triangle B onto triangle C.

Answer(a)(iii) ..... [2]

(b) Rotation through 90° anticlockwise about the origin is represented by the matrix $\mathbf{M} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

(i) Find $\mathbf{M}^{-1}$, the inverse of matrix $\mathbf{M}$.

Answer(b)(i) $\mathbf{M}^{-1} = \begin{pmatrix} & \\ & \end{pmatrix}$ [2]

(ii) Describe fully the **single** transformation represented by the matrix $\mathbf{M}^{-1}$.

Answer(b)(ii) ..... [2]

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[Turn over

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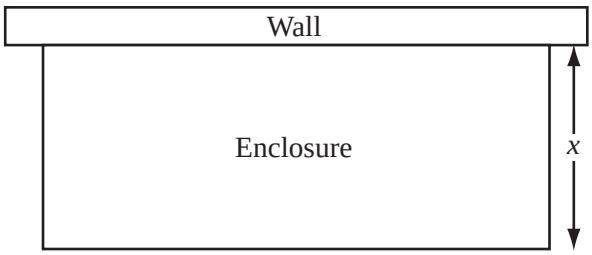
3



NOT TO SCALE

A farmer makes a rectangular enclosure for his animals.
He uses a wall for one side and a total of 72 metres of fencing for the other three sides.

The enclosure has width x metres and area A square metres.

(a) Show that $A = 72x - 2x^2$.

Answer (a)

[2]

(b) Factorise completely $72x - 2x^2$.

Answer(b) ..... [2]

(c) Complete the table for $A = 72x - 2x^2$.

x	0	5	10	15	20	25	30	35
A	0	310	520			550	360	

[3]

(d) Draw the graph of $A = 72x - 2x^2$ for $0 \leqslant x \leqslant 35$ on the grid opposite.

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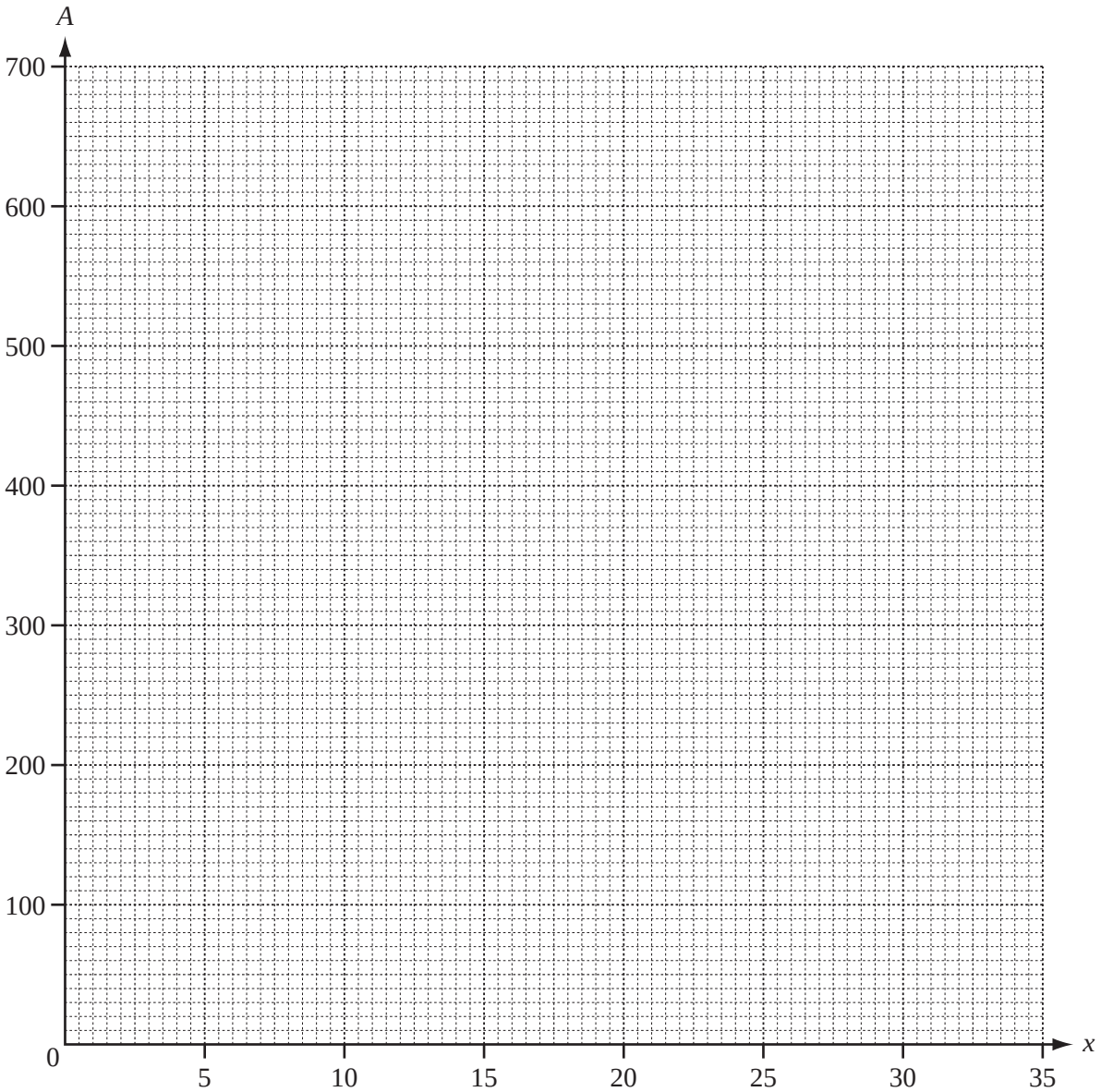

[4]

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(e) Use your graph to find

(i) the values of x when $A = 450$,

Answer(e)(i) $x =$ ..... or $x =$ ..... [2]

(ii) the maximum area of the enclosure.

Answer(e)(ii) ..... m^2 [1]

(f) Each animal must have at least 12 m^2 for grazing.

Calculate the greatest number of animals that the farmer can keep in an enclosure which has an area of 500 m^2.

Answer(f) ..... [2]

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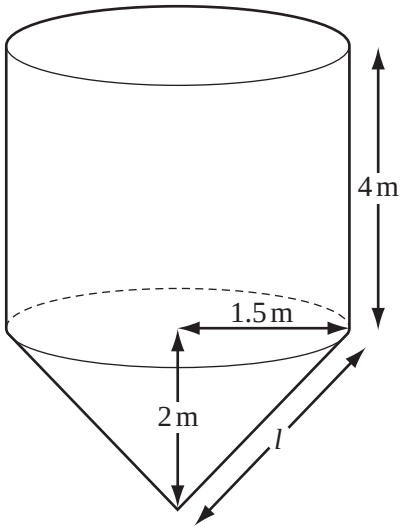
4



NOT TO SCALE

An **open** water storage tank is in the shape of a cylinder on top of a cone.
The radius of both the cylinder and the cone is 1.5 m.
The height of the cylinder is 4 m and the height of the cone is 2 m.

(a) Calculate the **total** surface area of the **outside** of the tank.
[The curved surface area, A, of a cone with radius r and slant height l is $A = \pi rl$.]

Answer(a) ..... m^2 [6]

(b) The tank is completely full of water.

(i) Calculate the volume of water in the tank and show that it rounds to 33 m^3, correct to the nearest whole number.

[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

Answer(b)(i)

[4]

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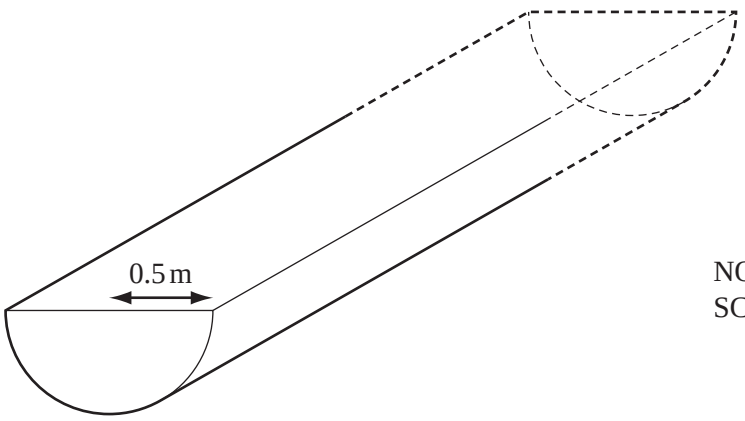
(ii)



The cross-section of an irrigation channel is a semi-circle of radius 0.5 m.
The 33 m^3 of water from the tank completely fills the irrigation channel.

Calculate the length of the channel.

Answer(b)(ii) ..... m [3]

(c) (i) Calculate the number of litres in a full tank of 33 m^3.

Answer(c)(i) ..... litres [1]

(ii) The water drains from the tank at a rate of 1800 litres per minute.

Calculate the time, in minutes and seconds, taken to empty the tank.

Answer(c)(ii) ..... min ..... s [2]

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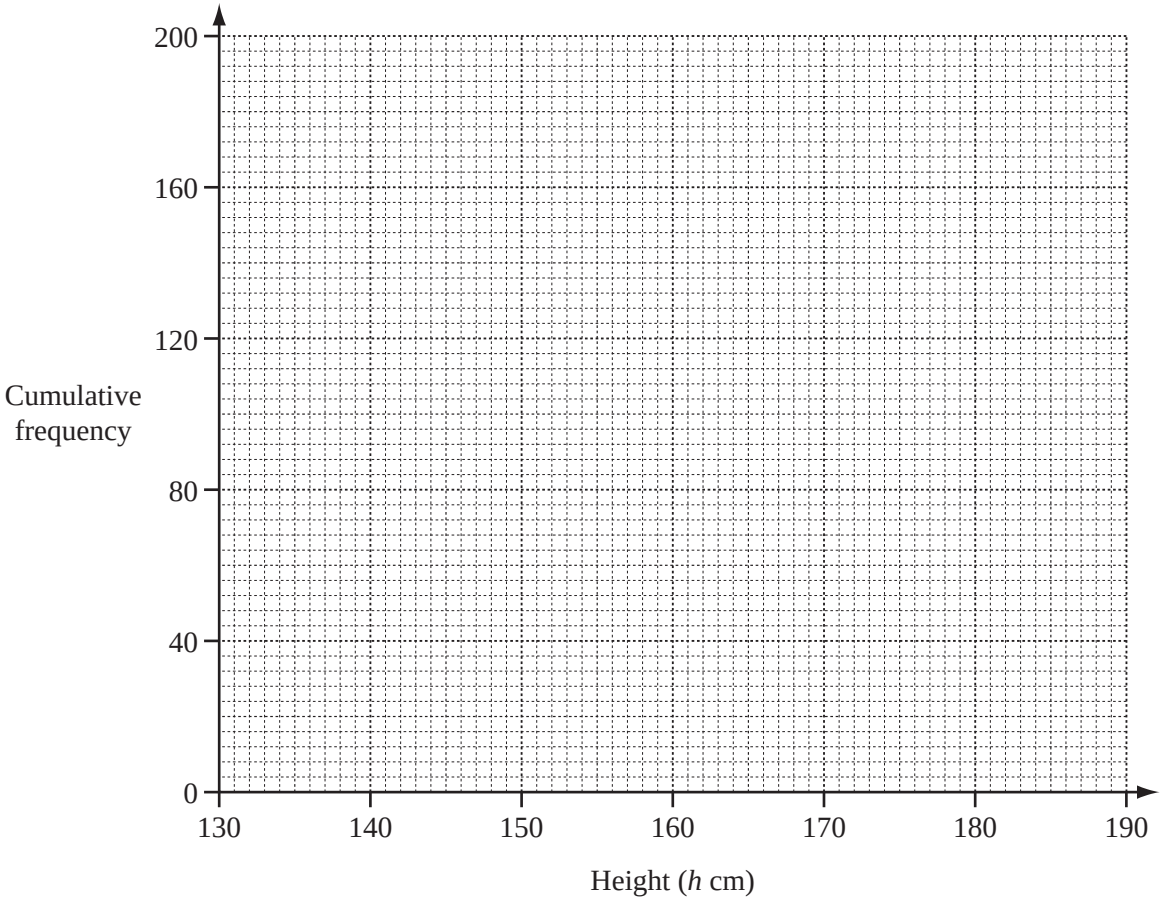
5 The cumulative frequency table shows the distribution of heights, h centimetres, of 200 students.

Height (h cm)	$\leqslant 130$	$\leqslant 140$	$\leqslant 150$	$\leqslant 160$	$\leqslant 165$	$\leqslant 170$	$\leqslant 180$	$\leqslant 190$
Cumulative frequency	0	10	50	95	115	145	180	200

(a) Draw a cumulative frequency diagram to show the information in the table.



[4]

(b) Use your diagram to find

(i) the median,

Answer(b)(i) ..... cm [1]

(ii) the upper quartile,

Answer(b)(ii) ..... cm [1]

(iii) the interquartile range.

Answer(b)(iii) ..... cm [1]

(c) **(i)** One of the 200 students is chosen at random.

Use the table to find the probability that the height of this student is greater than 170 cm. Give your answer as a fraction.

Answer(c)(i) ..... [1]

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(ii) One of the 200 students is chosen at random and then a second student is chosen at random from the remaining students.

Calculate the probability that one has a height greater than 170 cm and the other has a height of 140 cm or less.
Give your answer as a fraction.

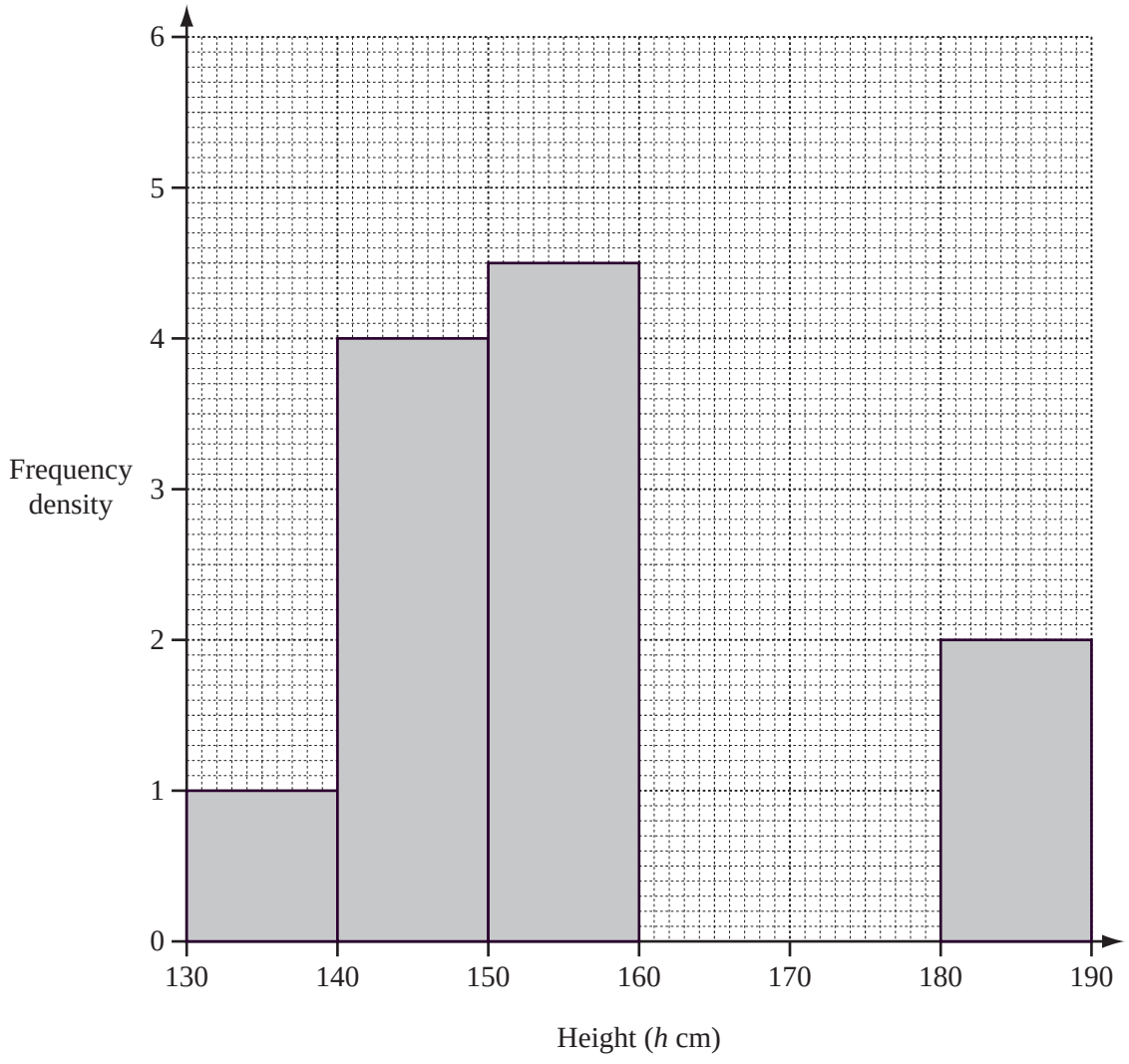
Answer(c)(ii) ..... [3]

(d) (i) Complete this frequency table which shows the distribution of the heights of the 200 students.

Height (h cm)	$130<h\leqslant140$	$140<h\leqslant150$	$150<h\leqslant160$	$160<h\leqslant165$	$165<h\leqslant170$	$170<h\leqslant180$	$180<h\leqslant190$
Frequency	10	40	45	20			

[2]

(ii) Complete this histogram to show the distribution of the heights of the 200 students.



[3]

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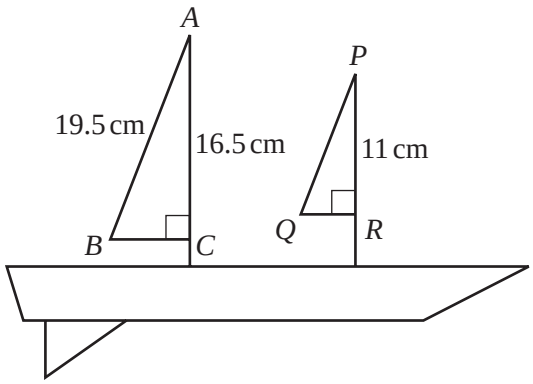
6 (a)



NOT TO SCALE

The diagram shows a toy boat.
$AC = 16.5$ cm, $AB = 19.5$ cm and $PR = 11$ cm.
Triangles ABC and PQR are **similar**.

(i) Calculate PQ.

Answer(a)(i) $PQ =$ ..... cm [2]

(ii) Calculate BC.

Answer(a)(ii) $BC =$ ..... cm [3]

(iii) Calculate angle ABC.

Answer(a)(iii) Angle $ABC =$ ..... [2]

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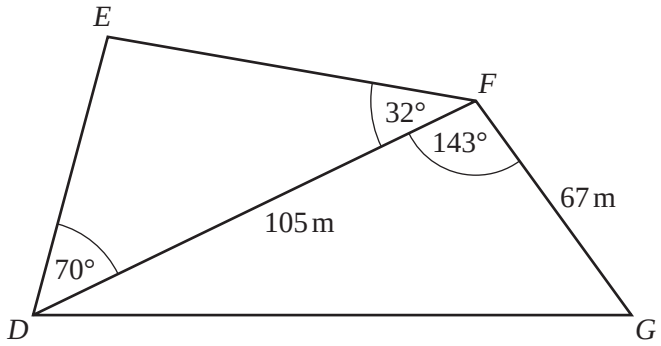
(iv) The toy boat is mathematically similar to a real boat.
The length of the real boat is 32 times the length of the toy boat.
The fuel tank in the toy boat holds 0.02 litres of diesel.

Calculate how many litres of diesel the fuel tank of the real boat holds.

Answer(a)(iv) ..... litres [2]

(b)



NOT TO SCALE

The diagram shows a field *DEFG*, in the shape of a quadrilateral, with a footpath along the diagonal *DF*.
DF = 105 m and *FG* = 67 m.
Angle *EDF* = 70°, angle *EFD* = 32° and angle *DFG* = 143°.

(i) Calculate *DG*.

Answer(b)(i) *DG* = ... m [4]

(ii) Calculate *EF*.

Answer(b)(ii) *EF* = ... m [4]

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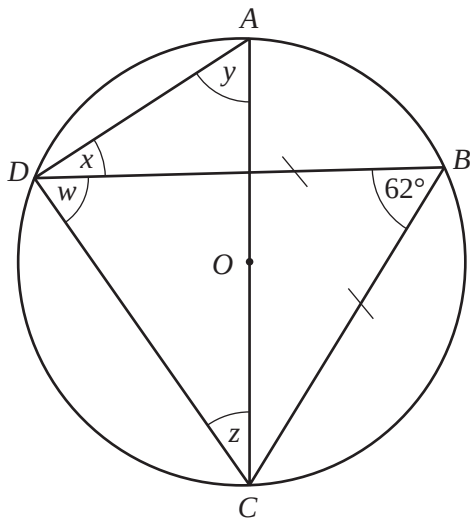
7 (a)



NOT TO SCALE

A, B, C and D are points on the circumference of a circle centre O.
AC is a diameter.
$BD = BC$ and angle $DBC = 62°$.

Work out the values of w, x, y and z.
Give a reason for each of your answers.

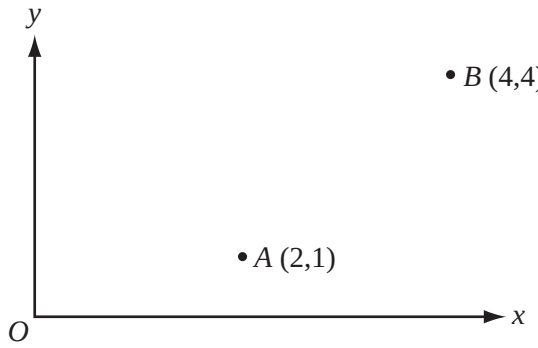
$w =$ ..... because ..... [2]

$x =$ ..... because ..... [2]

$y =$ ..... because ..... [2]

$z =$ ..... because ..... [2]

(b)



NOT TO SCALE

(i) Write down $\overrightarrow{AB}$ as a column vector.

Answer(b)(i) $\overrightarrow{AB} =$ $\begin{pmatrix} \\ \end{pmatrix}$ [1]

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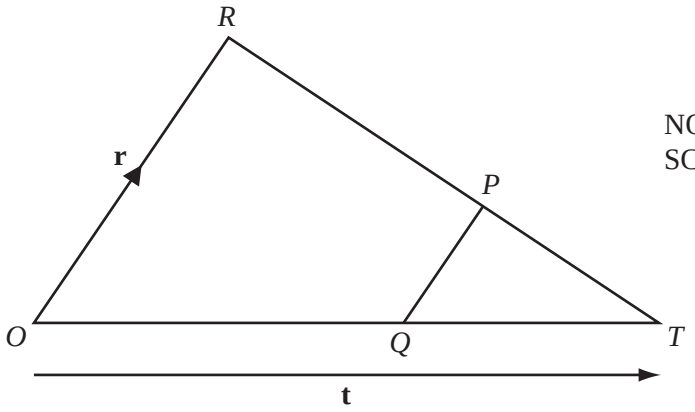
(ii) $\overrightarrow{AC} = \begin{pmatrix} 0 \\ 7 \end{pmatrix}$.

Work out $\overrightarrow{BC}$ as a column vector.

Answer(b)(ii) $\overrightarrow{BC} =$ $\begin{pmatrix} \\ \\ \end{pmatrix}$ [2]

(c)



NOT TO SCALE

$\overrightarrow{OR} = \mathbf{r}$ and $\overrightarrow{OT} = \mathbf{t}$.
P is on RT such that $RP : PT = 2 : 1$.
Q is on OT such that $OQ = \frac{2}{3} OT$.

Write the following in terms of **r** and/or **t**.
Simplify your answers where possible.

(i) $\overrightarrow{QT}$

Answer(c)(i) $\overrightarrow{QT} =$ ..... [1]

(ii) $\overrightarrow{TP}$

Answer(c)(ii) $\overrightarrow{TP} =$ ..... [2]

(iii) $\overrightarrow{QP}$

Answer(c)(iii) $\overrightarrow{QP} =$ ..... [2]

(iv) Write down two conclusions you can make about the line segment QP.

Answer(c)(iv) .....

..... [2]

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8 **(a)** $f(x) = 2x - 1$ $g(x) = x^2$

Work out

(i) f(2),

Answer(a)(i) ..... [1]

(ii) g(– 2),

Answer(a)(ii) ..... [1]

(iii) ff(x) in its simplest form,

Answer(a)(iii) ff(x) = ... [2]

(iv) $f^{-1}(x)$, the inverse of f(x),

Answer(a)(iv) $f^{-1}(x)$ = ... [2]

(v) x when gf(x) = 4.

Answer(a)(v) x = ... or x = ... [4]

(b) y is **inversely** proportional to x and $y = 8$ when $x = 2$.

Find,

(i) an equation connecting y and x,

Answer(b)(i) ..... [2]

(ii) y when $x = \frac{1}{2}$.

Answer(b)(ii) y = ... [1]

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9 **(a)** The first five terms P_1, P_2, P_3, P_4 and P_5 of a sequence are given below.

$1 = 1 = P_1$

$1 + 2 = 3 = P_2$

$1 + 2 + 3 = 6 = P_3$

$1 + 2 + 3 + 4 = 10 = P_4$

$1 + 2 + 3 + 4 + 5 = 15 = P_5$

(i) Write down the next term, P_6, in the sequence 1, 3, 6, 10, 15…

Answer(a)(i) ..... [1]

(ii) The formula for the nth term of this sequence is

$$P_n = \frac{1}{2}n(n + 1).$$

Show this formula is true when $n = 6$.

Answer (a)(ii)

[1]

(iii) Use the formula to find P_{50}, the 50th term of this sequence.

Answer(a)(iii) ..... [1]

(iv) Use your answer to **part (iii)** to find 3 + 6 + 9 + 12 +15 + …… + 150.

Answer(a)(iv) ..... [1]

(v) Find 1 + 2 + 3 + 4 + 5 +…… + 150.

Answer(a)(v) ..... [1]

(vi) Use your answers to **parts (iv)** and **(v)** to find the sum of the numbers less than 150 which are **not** multiples of 3.

Answer(a)(vi) ..... [1]

This question continues on the next page.

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(b) The first five terms, S_1, S_2, S_3, S_4 and S_5 of a different sequence are given below.

(1×1) $= 1 = S_1$

$(1 \times 2) + (2 \times 1)$ $= 4 = S_2$

$(1 \times 3) + (2 \times 2) + (3 \times 1)$ $= 10 = S_3$

$(1 \times 4) + (2 \times 3) + (3 \times 2) + (4 \times 1)$ $= 20 = S_4$

$(1 \times 5) + (2 \times 4) + (3 \times 3) + (4 \times 2) + (5 \times 1)$ $= 35 = S_5$

(i) Work out the next term, S_6, in the sequence 1, 4, 10, 20, 35…

Answer(b)(i) ..... [2]

(ii) The formula for the nth term of this sequence is

$$S_n = \frac{1}{6}n(n+1)(n+2).$$

Show this formula is true for $n = 6$.

Answer(b)(ii)

[1]

(iii) Find $(1 \times 20) + (2 \times 19) + (3 \times 18)$ …… $+ (20 \times 1)$.

Answer(b)(iii) ..... [1]

(c) Show that $S_6 - S_5 = P_6$, where P_6 is your answer to **part (a)(i)**.

Answer(c)

[1]

(d) Show by algebra that $S_n - S_{n-1} = P_n$. $\left[P_n = \frac{1}{2}n(n+1)\right]$

Answer(d)

[3]

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