Gradients, Tangents & Normals

Question Paper 2

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Differentiation
Sub Topic	Gradients, Tangents & Normals
Booklet	Question Paper 2

Time Allowed: **60 minutes**

Score: **/50**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 A straight line has equation $y = -2x + k$, where k is a constant, and a curve has equation $y = \frac{2}{x-3}$.

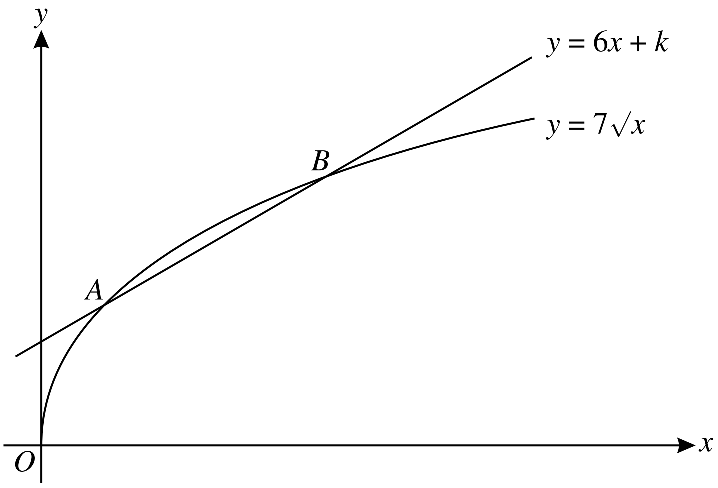
(i) Show that the x-coordinates of any points of intersection of the line and curve are given by the equation $2x^2 - (6 + k)x + (2 + 3k) = 0$. [1]

(ii) Find the two values of k for which the line is a tangent to the curve. [3]

The two tangents, given by the values of k found in part **(ii)**, touch the curve at points A and B.

(iii) Find the coordinates of A and B and the equation of the line AB. [6]

2



The diagram shows the curve $y = 7\sqrt{x}$ and the line $y = 6x + k$, where k is a constant. The curve and the line intersect at the points A and B.

(i) For the case where $k = 2$, find the x-coordinates of A and B. [4]

(ii) Find the value of k for which $y = 6x + k$ is a tangent to the curve $y = 7\sqrt{x}$. [2]

3 A curve has equation $y = 3x^3 - 6x^2 + 4x + 2$. Show that the gradient of the curve is never negative. [3]

4 The equation of a curve is $y^2 + 2x = 13$ and the equation of a line is $2y + x = k$, where k is a constant.

(i) In the case where $k = 8$, find the coordinates of the points of intersection of the line and the curve. [4]

(ii) Find the value of k for which the line is a tangent to the curve. [3]

5 A curve is such that $\frac{\mathrm{d}y}{\mathrm{d}x} = 5 - \frac{8}{x^2}$. The line $3y + x = 17$ is the normal to the curve at the point P on the curve. Given that the x-coordinate of P is positive, find

(i) the coordinates of P, [4]

(ii) the equation of the curve. [4]

6 **(i)** A straight line passes through the point (2, 0) and has gradient m. Write down the equation of the line. [1]

(ii) Find the two values of m for which the line is a tangent to the curve $y = x^2 - 4x + 5$. For each value of m, find the coordinates of the point where the line touches the curve. [6]

(iii) Express $x^2 - 4x + 5$ in the form $(x + a)^2 + b$ and hence, or otherwise, write down the coordinates of the minimum point on the curve. [2]

7 A curve is such that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{(1 + 2x)^2}$ and the point $(1, \frac{1}{2})$ lies on the curve.

(i) Find the equation of the curve. [4]

(ii) Find the set of values of x for which the gradient of the curve is less than $\frac{1}{3}$. [3]