

Gradients, Tangents & Normals

Question Paper 4

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Differentiation
Sub Topic	Gradients, Tangents & Normals
Booklet	Question Paper 4

Time Allowed: 60 minutes

Score: /50

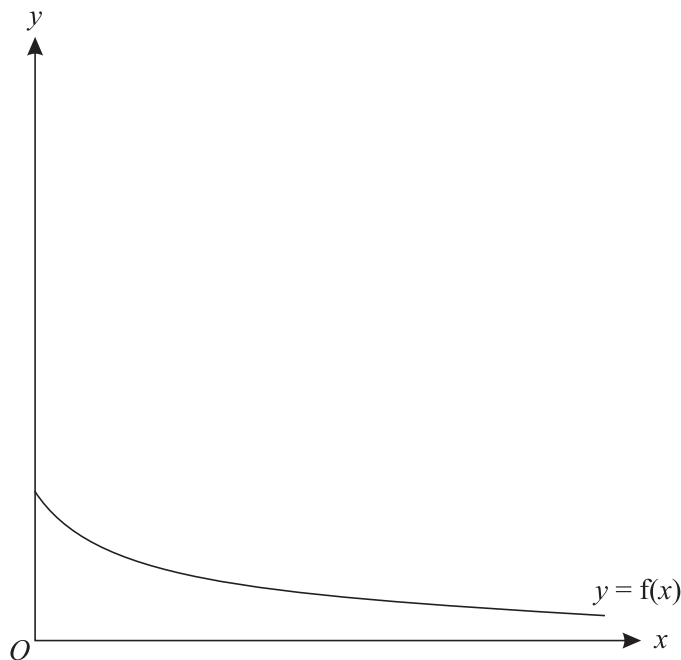
Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	77.5%	70%	62.5%	57.5%	45%	<45%

- 1 The equation of a curve is $y = 5 - \frac{8}{x}$.
- (i) Show that the equation of the normal to the curve at the point $P(2, 1)$ is $2y + x = 4$. [4]
- This normal meets the curve again at the point Q .
- (ii) Find the coordinates of Q . [3]
- (iii) Find the length of PQ . [2]
- 2 The function f is such that $f(x) = (3x + 2)^3 - 5$ for $x \geq 0$.
- (i) Obtain an expression for $f'(x)$ and hence explain why f is an increasing function. [3]
- (ii) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1} . [4]
- 3 A curve is such that $\frac{dy}{dx} = 4 - x$ and the point $P(2, 9)$ lies on the curve. The normal to the curve at P meets the curve again at Q . Find
- (i) the equation of the curve, [3]
- (ii) the equation of the normal to the curve at P , [3]
- (iii) the coordinates of Q . [3]
- 4 Find the value of the constant c for which the line $y = 2x + c$ is a tangent to the curve $y^2 = 4x$. [4]

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The diagram shows the graph of $y = f(x)$, where $f : x \mapsto \frac{6}{2x+3}$ for $x \geq 0$.

- (i) Find an expression, in terms of x , for $f'(x)$ and explain how your answer shows that f is a decreasing function. [3]
- (ii) Find an expression, in terms of x , for $f^{-1}(x)$ and find the domain of f^{-1} . [4]
- (iii) Copy the diagram and, on your copy, sketch the graph of $y = f^{-1}(x)$, making clear the relationship between the graphs. [2]

The function g is defined by $g : x \mapsto \frac{1}{2}x$ for $x \geq 0$.

- (iv) Solve the equation $fg(x) = \frac{3}{2}$. [3]

6 A curve is such that $\frac{dy}{dx} = \frac{4}{\sqrt{6-2x}}$, and $P(1, 8)$ is a point on the curve.

- (i) The normal to the curve at the point P meets the coordinate axes at Q and at R . Find the coordinates of the mid-point of QR . [5]
- (ii) Find the equation of the curve. [4]