

Rates of Change

Question Paper

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Differentiation
Sub Topic	Rates of Change
Booklet	Question Paper

Time Allowed: **47 minutes**

Score: **/39**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 A curve has equation $y = \frac{12}{3 - 2x}$.

(i) Find $\frac{dy}{dx}$. [2]

A point moves along this curve. As the point passes through A, the x -coordinate is increasing at a rate of 0.15 units per second and the y -coordinate is increasing at a rate of 0.4 units per second.

(ii) Find the possible x -coordinates of A. [4]

2 (a) The functions f and g are defined for $x \geq 0$ by

$$f : x \mapsto (ax + b)^{\frac{1}{3}}, \text{ where } a \text{ and } b \text{ are positive constants,}$$

$$g : x \mapsto x^2.$$

Given that $fg(1) = 2$ and $gf(9) = 16$,

(i) calculate the values of a and b , [4]

(ii) obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1} . [4]

(b) A point P travels along the curve $y = (7x^2 + 1)^{\frac{1}{3}}$ in such a way that the x -coordinate of P at time t minutes is increasing at a constant rate of 8 units per minute. Find the rate of increase of the y -coordinate of P at the instant when P is at the point $(3, 4)$. [5]

3 The equation of a curve is $y = 4\sqrt{x} + \frac{2}{\sqrt{x}}$.

(i) Obtain an expression for $\frac{dy}{dx}$. [3]

(ii) A point is moving along the curve in such a way that the x -coordinate is increasing at a constant rate of 0.12 units per second. Find the rate of change of the y -coordinate when $x = 4$. [2]

4 A curve is such that $\frac{d^2y}{dx^2} = -4x$. The curve has a maximum point at $(2, 12)$.

(i) Find the equation of the curve. [6]

A point P moves along the curve in such a way that the x -coordinate is increasing at 0.05 units per second.

(ii) Find the rate at which the y -coordinate is changing when $x = 3$, stating whether the y -coordinate is increasing or decreasing. [2]

5 The equation of a curve is $y = \frac{12}{x^2 + 3}$.

(i) Obtain an expression for $\frac{dy}{dx}$. [2]

(ii) Find the equation of the normal to the curve at the point $P(1, 3)$. [3]

(iii) A point is moving along the curve in such a way that the x -coordinate is increasing at a constant rate of 0.012 units per second. Find the rate of change of the y -coordinate as the point passes through P . [2]