

Stationary Points

Question Paper 2

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Differentiation
Sub Topic	Stationary Points
Booklet	Question Paper 2

Time Allowed: 63 minutes

Score: /52

Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

- 1 The equation of a curve is such that $\frac{d^2y}{dx^2} = 2x - 1$. Given that the curve has a minimum point at $(3, -10)$, find the coordinates of the maximum point. [8]

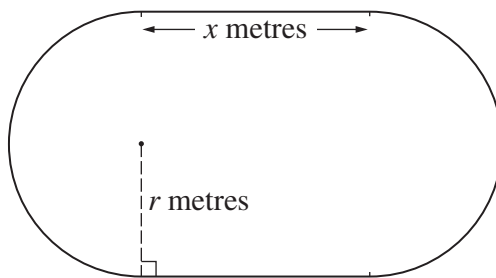
- 2 The base of a cuboid has sides of length x cm and $3x$ cm. The volume of the cuboid is 288 cm^3 .

- (i) Show that the total surface area of the cuboid, $A \text{ cm}^2$, is given by

$$A = 6x^2 + \frac{768}{x}. \quad [3]$$

- (ii) Given that x can vary, find the stationary value of A and determine its nature. [5]

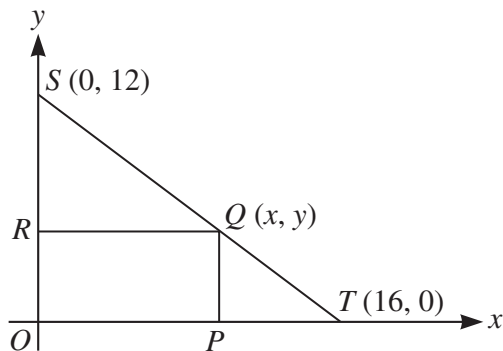
3



The inside lane of a school running track consists of two straight sections each of length x metres, and two semicircular sections each of radius r metres, as shown in the diagram. The straight sections are perpendicular to the diameters of the semicircular sections. The perimeter of the inside lane is 400 metres.

- (i) Show that the area, $A \text{ m}^2$, of the region enclosed by the inside lane is given by $A = 400r - \pi r^2$. [4]
- (ii) Given that x and r can vary, show that, when A has a stationary value, there are no straight sections in the track. Determine whether the stationary value is a maximum or a minimum. [5]

4



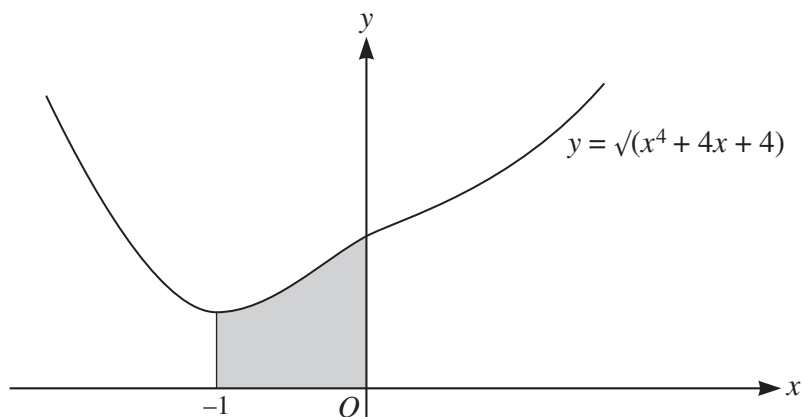
In the diagram, S is the point $(0, 12)$ and T is the point $(16, 0)$. The point Q lies on ST , between S and T , and has coordinates (x, y) . The points P and R lie on the x -axis and y -axis respectively and $OPQR$ is a rectangle.

(i) Show that the area, A , of the rectangle $OPQR$ is given by $A = 12x - \frac{3}{4}x^2$. [3]

(ii) Given that x can vary, find the stationary value of A and determine its nature. [4]

5 A curve has equation $y = \frac{k^2}{x+2} + x$, where k is a positive constant. Find, in terms of k , the values of x for which the curve has stationary points and determine the nature of each stationary point. [8]

6



The diagram shows the curve $y = \sqrt{x^4 + 4x + 4}$.

(i) Find the equation of the tangent to the curve at the point $(0, 2)$. [4]

(ii) Show that the x -coordinates of the points of intersection of the line $y = x + 2$ and the curve are given by the equation $(x + 2)^2 = x^4 + 4x + 4$. Hence find these x -coordinates. [4]

(iii) The region shaded in the diagram is rotated through 360° about the x -axis. Find the volume of revolution. [4]