Stationary Points

Question Paper 3

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Differentiation
Sub Topic	Stationary Points
Booklet	Question Paper 3

Time Allowed: **62 minutes**

Score: **/51**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 A curve has equation $y = \mathrm{f}(x)$ and is such that $\mathrm{f}'(x) = 3x^{\frac{1}{2}} + 3x^{-\frac{1}{2}} - 10$.

(i) By using the substitution $u = x^{\frac{1}{2}}$, or otherwise, find the values of x for which the curve $y = \mathrm{f}(x)$ has stationary points. [4]

(ii) Find $\mathrm{f}''(x)$ and hence, or otherwise, determine the nature of each stationary point. [3]

(iii) It is given that the curve $y = \mathrm{f}(x)$ passes through the point $(4, -7)$. Find $\mathrm{f}(x)$. [4]

2 The non-zero variables x, y and u are such that $u = x^2y$. Given that $y + 3x = 9$, find the stationary value of u and determine whether this is a maximum or a minimum value. [7]

3 A curve has equation $y = 2x + \dfrac{1}{(x-1)^2}$. Verify that the curve has a stationary point at $x = 2$ and determine its nature. [5]

4 A curve is such that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2(3x+4)^{\frac{3}{2}} - 6x - 8.$$

(i) Find $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$. [2]

(ii) Verify that the curve has a stationary point when $x = -1$ and determine its nature. [2]

(iii) It is now given that the stationary point on the curve has coordinates $(-1, 5)$. Find the equation of the curve. [5]

5 It is given that a curve has equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = x^3 - 2x^2 + x$.

(i) Find the set of values of x for which the gradient of the curve is less than 5. [4]

(ii) Find the values of $\mathrm{f}(x)$ at the two stationary points on the curve and determine the nature of each stationary point. [5]

6 The function f is such that $f(x) = 8 - (x - 2)^2$, for $x \in \mathbb{R}$.

(i) Find the coordinates and the nature of the stationary point on the curve $y = f(x)$. [3]

The function g is such that $g(x) = 8 - (x - 2)^2$, for $k \leqslant x \leqslant 4$, where k is a constant.

(ii) State the smallest value of k for which g has an inverse. [1]

For this value of k,

(iii) find an expression for $g^{-1}(x)$, [3]

(iv) sketch, on the same diagram, the graphs of $y = g(x)$ and $y = g^{-1}(x)$. [3]