Trigonometry

Question Paper 2

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Trigonometry
Sub Topic	
Booklet	Question Paper 2

Time Allowed: **59 minutes**

Score: **/49**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 **(i)** Express $3\cos\theta + \sin\theta$ in the form $\boldsymbol{R}\cos(\theta - \alpha)$, where $\boldsymbol{R} > 0$ and $0° < \alpha < 90°$, giving the exact value of $\boldsymbol{R}$ and the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$3\cos 2\boldsymbol{x} + \sin 2\boldsymbol{x} = 2,$$

giving all solutions in the interval $0° \leqslant \boldsymbol{x} \leqslant 360°$. [5]

2 **(i)** Express $5\sin 2\theta + 2\cos 2\theta$ in the form $\boldsymbol{R}\sin(2\theta + \alpha)$, where $\boldsymbol{R} > 0$ and $0° < \alpha < 90°$, giving the exact value of $\boldsymbol{R}$ and the value of α correct to 2 decimal places. [3]

Hence

(ii) solve the equation

$$5\sin 2\theta + 2\cos 2\theta = 4,$$

giving all solutions in the interval $0° \leqslant \theta \leqslant 360°$, [5]

(iii) determine the least value of $\dfrac{1}{(10\sin 2\theta + 4\cos 2\theta)^2}$ as θ varies. [2]

3 **(i)** Prove the identity

$$\frac{1}{\sin(\boldsymbol{x} - 60°) + \cos(\boldsymbol{x} - 30°)} \equiv \operatorname{cosec} \boldsymbol{x}.$$ [3]

(ii) Hence solve the equation

$$\frac{2}{\sin(\boldsymbol{x} - 60°) + \cos(\boldsymbol{x} - 30°)} = 3\cot^2 \boldsymbol{x} - 2,$$

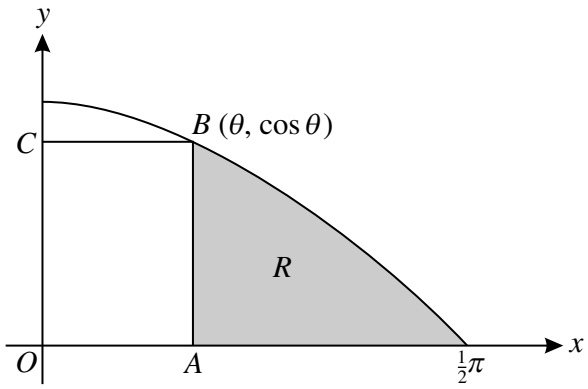
for $0° < \boldsymbol{x} < 360°$. [6]

4 Solve the equation

$$2\cos 2\theta = 4\cos\theta - 3,$$

for $0° \leqslant \theta \leqslant 180°$. [4]

5



The diagram shows the curve $y = \cos x$, for $0 \leqslant x \leqslant \frac{1}{2}\pi$. A rectangle $OABC$ is drawn, where B is the point on the curve with x-coordinate θ, and A and C are on the axes, as shown. The shaded region R is bounded by the curve and by the lines $x = \theta$ and $y = 0$.

(i) Find the area of R in terms of θ. [2]

(ii) The area of the rectangle $OABC$ is equal to the area of R. Show that

$$\theta = \frac{1 - \sin\theta}{\cos\theta}.$$ [1]

(iii) Use the iterative formula $\theta_{n+1} = \dfrac{1 - \sin\theta_n}{\cos\theta_n}$, with initial value $\theta_1 = 0.5$, to determine the value of θ correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

6 **(i)** By differentiating $\frac{1}{\cos\theta}$, show that if $y = \sec\theta$ then $\frac{\mathrm{d}y}{\mathrm{d}\theta} = \tan\theta\sec\theta$. [3]

(ii) Hence show that

$$\frac{\mathrm{d}^2y}{\mathrm{d}\theta^2} = a\sec^3\theta + b\sec\theta,$$

giving the values of a and b. [4]

(iii) Find the exact value of

$$\int_0^{\frac{1}{4}\pi} (1 + \tan^2\theta - 3\sec\theta\tan\theta)\,\mathrm{d}\theta.$$ [5]