Trigonometry

Question Paper 3

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Trigonometry
Sub Topic	
Booklet	Question Paper 3

Time Allowed: **60 minutes**

Score: **/50**

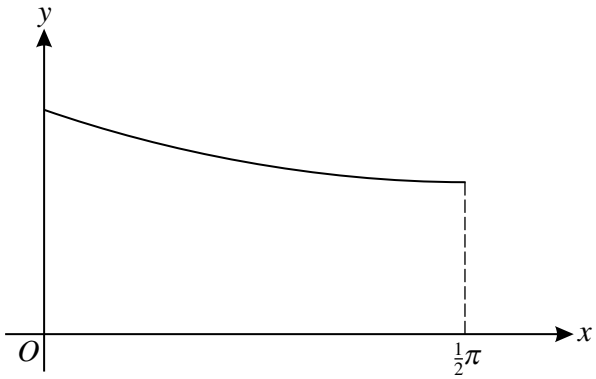
Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 The curve with equation $y = \dfrac{\sin 2x}{e^{2x}}$ has one stationary point in the interval $0 \leqslant x \leqslant \frac{1}{2}\pi$. Find the exact x-coordinate of this point. [4]

2



The diagram shows the part of the curve $y = \sqrt{(2 - \sin x)}$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$.

(i) Use the trapezium rule with 2 intervals to estimate the value of

$$\int_0^{\frac{1}{2}\pi} \sqrt{(2 - \sin x)}\, \mathrm{d}x,$$

giving your answer correct to 2 decimal places. [3]

(ii) The line $y = x$ intersects the curve $y = \sqrt{(2 - \sin x)}$ at the point P. Use the iterative formula

$$x_{n+1} = \sqrt{(2 - \sin x_n)}$$

to determine the x-coordinate of P correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

3 **(a)** Given that $\tan A = t$ and $\tan(A + B) = 4$, find $\tan B$ in terms of t. [3]

(b) Solve the equation

$$2\tan(45^\circ - x) = 3\tan x,$$

giving all solutions in the interval $0^\circ \leqslant x \leqslant 360^\circ$. [6]

4 **(i)** Given that $35 + \sec^2\theta = 12\tan\theta$, find the value of $\tan\theta$. [3]

(ii) Hence, showing the use of an appropriate formula in each case, find the exact value of

(a) $\tan(\theta - 45°)$, [2]

(b) $\tan 2\theta$. [2]

5 **(i)** By first expanding $\cos(2x + x)$, show that

$$\cos 3x \equiv 4\cos^3 x - 3\cos x.$$ [5]

(ii) Hence show that

$$\int_0^{\frac{1}{6}\pi} (2\cos^3 x - \cos x)\,\mathrm{d}x = \frac{5}{12}.$$ [5]

6 **(i)** By sketching a suitable pair of graphs, show that the equation

$$\frac{1}{x} = \sin x,$$

where x is in radians, has only one root for $0 < x \leqslant \frac{1}{2}\pi$. [2]

(ii) Verify by calculation that this root lies between $x = 1.1$ and $x = 1.2$. [2]

(iii) Use the iterative formula $x_{n+1} = \dfrac{1}{\sin x_n}$ to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 The parametric equations of a curve are

$$x = 1 + 2\sin^2\theta, \quad y = 4\tan\theta.$$

(i) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{\sin\theta\cos^3\theta}$. [3]

(ii) Find the equation of the tangent to the curve at the point where $\theta = \frac{1}{4}\pi$, giving your answer in the form $y = mx + c$. [4]