Trigonometry

Question Paper 4

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Trigonometry
Sub Topic	
Booklet	Question Paper 4

Time Allowed: **58 minutes**

Score: **/48**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 **(i)** Express $5\cos\theta - 3\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the exact value of R and the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$5\cos\theta - 3\sin\theta = 4,$$

giving all solutions in the interval $0° \leqslant \theta \leqslant 360°$. [4]

(iii) Write down the least value of $15\cos\theta - 9\sin\theta$ as θ varies. [1]

2 Solve the equation $5\sec^2 2\theta = \tan 2\theta + 9$, giving all solutions in the interval $0° \leqslant \theta \leqslant 180°$. [6]

3 **(i)** Express $4\sin\theta - 6\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the exact value of R and the value of α correct to 2 decimal places. [3]

(ii) Solve the equation $4\sin\theta - 6\cos\theta = 3$ for $0° \leqslant \theta \leqslant 360°$. [4]

(iii) Find the greatest and least possible values of $(4\sin\theta - 6\cos\theta)^2 + 8$ as θ varies. [2]

4 **(i)** Prove that $\sin^2 2\theta(\operatorname{cosec}^2\theta - \sec^2\theta) \equiv 4\cos 2\theta$. [3]

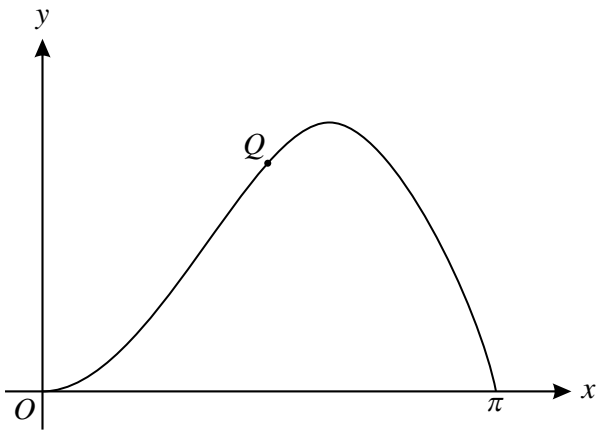
(ii) Hence

(a) solve for $0° \leqslant \theta \leqslant 180°$ the equation $\sin^2 2\theta(\operatorname{cosec}^2\theta - \sec^2\theta) = 3$, [4]

(b) find the exact value of $\operatorname{cosec}^2 15° - \sec^2 15°$. [2]

5 Solve the equation $8 + \cot\theta = 2\,\text{cosec}^2\theta$, giving all solutions in the interval $0° \leqslant \theta \leqslant 360°$. [6]

6



The diagram shows the curve $y = x\sin x$, for $0 \leqslant x \leqslant \pi$. The point $Q\left(\frac{1}{2}\pi, \frac{1}{2}\pi\right)$ lies on the curve.

(i) Show that the normal to the curve at Q passes through the point $(\pi, 0)$. [5]

(ii) Find $\dfrac{\mathrm{d}}{\mathrm{d}x}(\sin x - x\cos x)$. [2]

(iii) Hence evaluate $\displaystyle\int_0^{\frac{1}{2}\pi} x\sin x\,\mathrm{d}x$. [3]