

Trigonometry

Question Paper 5

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|------------|-----------------------|
| Level | International A Level |
| Subject | Maths |
| Exam Board | CIE |
| Topic | Trigonometry |
| Sub Topic | |
| Booklet | Question Paper 5 |

Time Allowed: **63 minutes**

Score: **/52**

Percentage: **/100**

Grade Boundaries:

| A* | A | B | C | D | E | U |
|------|--------|-----|-------|-------|-----|------|
| >85% | '77.5% | 70% | 62.5% | 57.5% | 45% | <45% |

- 1 (i) Express $2 \sin \theta - \cos \theta$ in the form $R \sin(\theta - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$, giving the exact value of R and the value of α correct to 2 decimal places. [3]

- (ii) Hence solve the equation

$$2 \sin \theta - \cos \theta = -0.4,$$

giving all solutions in the interval $0^\circ \leq \theta \leq 360^\circ$. [4]

- 2 (i) Show that the equation $\tan(x + 45^\circ) = 6 \tan x$ can be written in the form

$$6 \tan^2 x - 5 \tan x + 1 = 0. [3]$$

- (ii) Hence solve the equation $\tan(x + 45^\circ) = 6 \tan x$, for $0^\circ < x < 180^\circ$. [3]

- 3 (i) Prove the identity

$$\sin(x - 30^\circ) + \cos(x - 60^\circ) \equiv (\sqrt{3}) \sin x. [3]$$

- (ii) Hence solve the equation

$$\sin(x - 30^\circ) + \cos(x - 60^\circ) = \frac{1}{2} \sec x,$$

for $0^\circ < x < 360^\circ$. [6]

- 4 (i) Show that the equation $\sin(60^\circ - x) = 2 \sin x$ can be written in the form $\tan x = k$, where k is a constant. [4]

- (ii) Hence solve the equation $\sin(60^\circ - x) = 2 \sin x$, for $0^\circ < x < 360^\circ$. [2]

- 5 (i) Use the trapezium rule with two intervals to estimate the value of

$$\int_0^{\frac{1}{3}\pi} \sec x \, dx,$$

giving your answer correct to 2 decimal places. [3]

- (ii) Using a sketch of the graph of $y = \sec x$ for $0 \leq x \leq \frac{1}{3}\pi$, explain whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part (i). [2]

- 6 (i) Express $3 \cos x + 4 \sin x$ in the form $R \cos(x - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$, stating the exact value of R and giving the value of α correct to 2 decimal places. [3]

- (ii) Hence solve the equation

$$3 \cos x + 4 \sin x = 4.5,$$

giving all solutions in the interval $0^\circ < x < 360^\circ$. [4]

- 7 Solve the equation $\sec x = 4 - 2 \tan^2 x$, giving all solutions in the interval $0^\circ \leq x \leq 180^\circ$. [6]

- 8 (i) Show that the equation

$$\sin(x + 30^\circ) = 2 \cos(x + 60^\circ)$$

can be written in the form

$$(3\sqrt{3}) \sin x = \cos x. \quad [3]$$

- (ii) Hence solve the equation

$$\sin(x + 30^\circ) = 2 \cos(x + 60^\circ),$$

for $-180^\circ \leq x \leq 180^\circ$. [3]