

# Trigonometry

## Question Paper 6

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Trigonometry
Sub Topic	
Booklet	Question Paper 6

Time Allowed: **82 minutes**

Score: **/68**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

- 1 (i) Express  $5 \cos \theta - \sin \theta$  in the form  $R \cos(\theta + \alpha)$ , where  $R > 0$  and  $0^\circ < \alpha < 90^\circ$ , giving the exact value of  $R$  and the value of  $\alpha$  correct to 2 decimal places. [3]

- (ii) Hence solve the equation

$$5 \cos \theta - \sin \theta = 4,$$

giving all solutions in the interval  $0^\circ \leq \theta \leq 360^\circ$ . [4]

- 2 (i) Express  $8 \sin \theta - 15 \cos \theta$  in the form  $R \sin(\theta - \alpha)$ , where  $R > 0$  and  $0^\circ < \alpha < 90^\circ$ , giving the exact value of  $R$  and the value of  $\alpha$  correct to 2 decimal places. [3]

- (ii) Hence solve the equation

$$8 \sin \theta - 15 \cos \theta = 14,$$

giving all solutions in the interval  $0^\circ \leq \theta \leq 360^\circ$ . [4]

- 3 (i) Prove the identity

$$\tan(x + 45^\circ) - \tan(45^\circ - x) \equiv 2 \tan 2x. \quad [4]$$

- (ii) Hence solve the equation

$$\tan(x + 45^\circ) - \tan(45^\circ - x) = 2,$$

for  $0^\circ \leq x \leq 180^\circ$ . [3]

- 4 (i) Prove the identity

$$\cos(x + 30^\circ) + \sin(x + 60^\circ) \equiv (\sqrt{3}) \cos x. \quad [3]$$

- (ii) Hence solve the equation

$$\cos(x + 30^\circ) + \sin(x + 60^\circ) = 1,$$

for  $0^\circ < x < 90^\circ$ . [2]

- 5 (i) Express  $12 \cos \theta - 5 \sin \theta$  in the form  $R \cos(\theta + \alpha)$ , where  $R > 0$  and  $0^\circ < \alpha < 90^\circ$ , giving the exact value of  $R$  and the value of  $\alpha$  correct to 2 decimal places. [3]

- (ii) Hence solve the equation

$$12 \cos \theta - 5 \sin \theta = 10,$$

giving all solutions in the interval  $0^\circ \leq \theta \leq 360^\circ$ . [4]

- 6 Find the values of  $x$  satisfying the equation

$$3 \sin 2x = \cos x,$$

for  $0^\circ \leq x \leq 90^\circ$ . [4]

- 7 (i) Express  $3 \sin \theta + 4 \cos \theta$  in the form  $R \sin(\theta + \alpha)$ , where  $R > 0$  and  $0^\circ < \alpha < 90^\circ$ , giving the value of  $\alpha$  correct to 2 decimal places. [3]

- (ii) Hence solve the equation

$$3 \sin \theta + 4 \cos \theta = 4.5,$$

giving all solutions in the interval  $0^\circ \leq \theta \leq 360^\circ$ , correct to 1 decimal place. [4]

- (iii) Write down the least value of  $3 \sin \theta + 4 \cos \theta + 7$  as  $\theta$  varies. [1]

- 8 (i) Express  $\cos \theta + (\sqrt{3}) \sin \theta$  in the form  $R \cos(\theta - \alpha)$ , where  $R > 0$  and  $0 < \alpha < \frac{1}{2}\pi$ , giving the exact value of  $\alpha$ . [3]

- (ii) Hence show that one solution of the equation

$$\cos \theta + (\sqrt{3}) \sin \theta = \sqrt{2}$$

is  $\theta = \frac{7}{12}\pi$ , and find the other solution in the interval  $0 < \theta < 2\pi$ . [4]

- 9 (i) Show that the equation

$$\tan(45^\circ + x) = 4 \tan(45^\circ - x)$$

can be written in the form

$$3 \tan^2 x - 10 \tan x + 3 = 0. \quad [4]$$

- (ii) Hence solve the equation

$$\tan(45^\circ + x) = 4 \tan(45^\circ - x),$$

for  $0^\circ < x < 90^\circ$ . [3]

- 10 The angle  $x$ , measured in degrees, satisfies the equation

$$\cos(x - 30^\circ) = 3 \sin(x - 60^\circ).$$

- (i) By expanding each side, show that the equation may be simplified to

$$(2\sqrt{3}) \cos x = \sin x. \quad [3]$$

- (ii) Find the two possible values of  $x$  lying between  $0^\circ$  and  $360^\circ$ . [3]

- (iii) Find the exact value of  $\cos 2x$ , giving your answer as a fraction. [3]