Differentiation – Parametric, Implicit, Products & Quotients

Question Paper 3

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Differentiation
Sub Topic	Differentiation – Parametric, Implicit, Products & Quotients
Booklet	Question Paper 3

Time Allowed: **59 minutes**

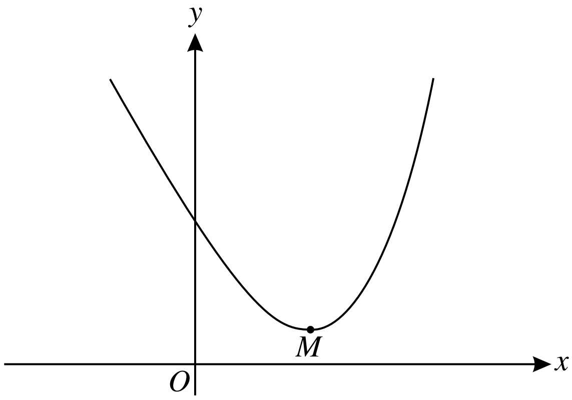
Score: **/49**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1



The diagram shows the curve $y = 4e^{\frac{1}{2}x} - 6x + 3$ and its minimum point M.

(i) Show that the x-coordinate of M can be written in the form $\ln a$, where the value of a is to be stated. [5]

(ii) Find the exact value of the area of the region enclosed by the curve and the lines $x = 0$, $x = 2$ and $y = 0$. [4]

2 A curve has parametric equations

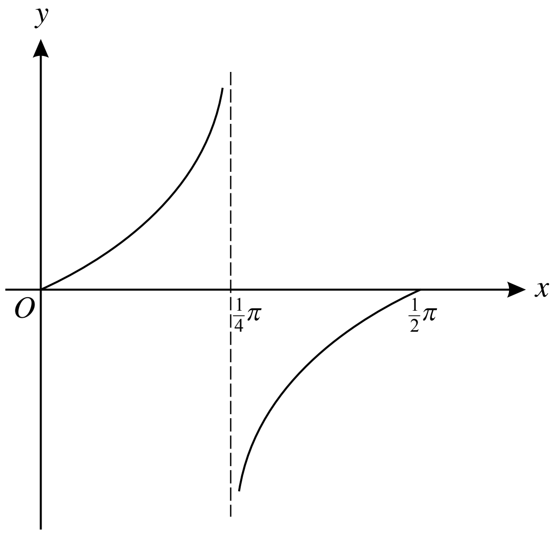
$$x = \frac{1}{(2t+1)^2}, \quad y = \sqrt{(t+2)}.$$

The point P on the curve has parameter p and it is given that the gradient of the curve at P is -1.

(i) Show that $p = (p+2)^{\frac{1}{6}} - \frac{1}{2}$. [6]

(ii) Use an iterative process based on the equation in part **(i)** to f nd the value of p correct to 3 decimal places. Use a starting value of 0.7 and show the result of each iteration to 5 decimal places. [3]

3



The diagram shows the part of the curve $y = \frac{1}{2}\tan 2x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$. Find the x-coordinates of the points on this part of the curve at which the gradient is 4. [5]

4 The parametric equations of a curve are

$$x = e^{3t}, \quad y = t^2 e^t + 3.$$

(i) Show that $\frac{dy}{dx} = \frac{t(t+2)}{3e^{2t}}$. [4]

(ii) Show that the tangent to the curve at the point $(1, 3)$ is parallel to the x-axis. [2]

(iii) Find the exact coordinates of the other point on the curve at which the tangent is parallel to the x-axis. [2]

5 Find the gradient of the curve $y = \ln(5x + 1)$ at the point where $x = 4$. [3]

6 The equation of a curve is $2x^2 - 3x - 3y + y^2 = 6$.

(i) Show that $\frac{dy}{dx} = \frac{4x - 3}{3 - 2y}$. [3]

(ii) Find the coordinates of the two points on the curve at which the gradient is -1. [6]