# Differentiation – Parametric, Implicit, Products & Quotients

## Question Paper 4

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| **Level** | International A Level |
| **Subject** | Maths |
| **Exam Board** | CIE |
| **Topic** | Differentiation |
| **Sub Topic** | Differentiation – Parametric, Implicit, Products & Quotients |
| **Booklet** | Question Paper 4 |

**Time Allowed:** **62 minutes**

**Score:** **/51**

**Percentage:** **/100**

**Grade Boundaries:**

| A* | A | B | C | D | E | U |
|---|---|---|---|---|---|---|
| >85% | '77.5% | 70% | 62.5% | 57.5% | 45% | <45% |

**1** A curve has parametric equations

$$x = 3t + \sin 2t, \quad y = 4 + 2\cos 2t.$$

Find the exact gradient of the curve at the point for which $t = \frac{1}{6}\pi$. [4]

**2** Find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x = 4$ in each of the following cases:

**(i)** $y = x\ln(x - 3)$, [4]

**(ii)** $y = \frac{x - 1}{x + 1}$. [3]

**3** A curve has equation $x^2 + 2y^2 + 5x + 6y = 10$. Find the equation of the tangent to the curve at the point $(2, -1)$. Give your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers. [6]

**4** The curve $y = 4x^2 \ln x$ has one stationary point.

**(i)** Find the coordinates of this stationary point, giving your answers correct to 3 decimal places. [5]

**(ii)** Determine whether this point is a maximum or a minimum point. [2]

**5** The parametric equations of a curve are

$$x = 1 + \ln(t - 2), \quad y = t + \frac{9}{t}, \quad \text{for } t > 2.$$

**(i)** Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(t^2 - 9)(t - 2)}{t^2}$. [3]

**(ii)** Find the coordinates of the only point on the curve at which the gradient is equal to 0. [3]

**6** The equation of a curve is

$$x^2y + y^2 = 6x.$$

**(i)** Show that $\dfrac{dy}{dx} = \dfrac{6 - 2xy}{x^2 + 2y}$. [4]

**(ii)** Find the equation of the tangent to the curve at the point with coordinates (1, 2), giving your answer in the form $ax + by + c = 0$. [3]

**7** The equation of a curve is $y = x^3e^{-x}$.

**(i)** Show that the curve has a stationary point where $x = 3$. [3]

**(ii)** Find the equation of the tangent to the curve at the point where $x = 1$. [4]

**8** The curve with equation $y = x \ln x$ has one stationary point.

**(i)** Find the exact coordinates of this point, giving your answers in terms of e. [5]

**(ii)** Determine whether this point is a maximum or a minimum point. [2]