Differentiation – Parametric, Implicit, Products & Quotients

Question Paper 5

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Differentiation
Sub Topic	Differentiation – Parametric, Implicit, Products & Quotients
Booklet	Question Paper 5

Time Allowed: **57 minutes**

Score: **/47**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 The equation of a curve is $y^2 + 2xy - x^2 = 2$.

(i) Find the coordinates of the two points on the curve where $x = 1$. [2]

(ii) Show by differentiation that at one of these points the tangent to the curve is parallel to the x-axis. Find the equation of the tangent to the curve at the other point, giving your answer in the form $ax + by + c = 0$. [7]

2 The parametric equations of a curve are

$$x = 1 - e^{-t}, \quad y = e^t + e^{-t}.$$

(i) Show that $\frac{dy}{dx} = e^{2t} - 1$. [3]

(ii) Hence f nd the exact value of t at the point on the curve at which the gradient is 2. [2]

3 The parametric equations of a curve are

$$x = 4\sin\theta, \quad y = 3 - 2\cos 2\theta,$$

where $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$. Express $\frac{dy}{dx}$ in terms of θ, simplifying your answer as far as possible. [5]

4 Find the exact coordinates of the point on the curve $y = xe^{-\frac{1}{2}x}$ at which $\frac{d^2y}{dx^2} = 0$. [7]

5 It is given that the curve $y = (x - 2)e^x$ has one stationary point.

(i) Find the exact coordinates of this point. [5]

(ii) Determine whether this point is a maximum or a minimum point. [2]

6 The equation of a curve is

$$x^2 + y^2 - 4xy + 3 = 0.$$

(i) Show that $\dfrac{dy}{dx} = \dfrac{2y - x}{y - 2x}$. [4]

(ii) Find the coordinates of each of the points on the curve where the tangent is parallel to the x-axis. [5]

7 The equation of a curve is $y = 2x - \tan x$, where x is in radians. Find the coordinates of the stationary points of the curve for which $-\frac{1}{2}\pi < x < \frac{1}{2}\pi$. [5]