Differentiation – Parametric, Implicit, Products & Quotients

Question Paper 6

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Differentiation
Sub Topic	Differentiation – Parametric, Implicit, Products & Quotients
Booklet	Question Paper 6

Time Allowed: **58 minutes**

Score: **/48**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 The parametric equations of a curve are

$$x = 3t + \ln(t - 1), \quad y = t^2 + 1, \quad \text{for } t > 1.$$

(i) Express $\frac{dy}{dx}$ in terms of t. [3]

(ii) Find the coordinates of the only point on the curve at which the gradient of the curve is equal to 1. [4]

2 The equation of a curve is $y = x + 2\cos x$. Find the x-coordinates of the stationary points of the curve for $0 \leqslant x \leqslant 2\pi$, and determine the nature of each of these stationary points. [7]

3 The equation of a curve is $3x^2 + 2xy + y^2 = 6$. It is given that there are two points on the curve where the tangent is parallel to the x-axis.

(i) Show by differentiation that, at these points, $y = -3x$. [4]

(ii) Hence find the coordinates of the two points. [4]

4 The equation of a curve is $x^3 + y^3 = 9xy$.

(i) Show that $\frac{dy}{dx} = \frac{3y - x^2}{y^2 - 3x}$. [4]

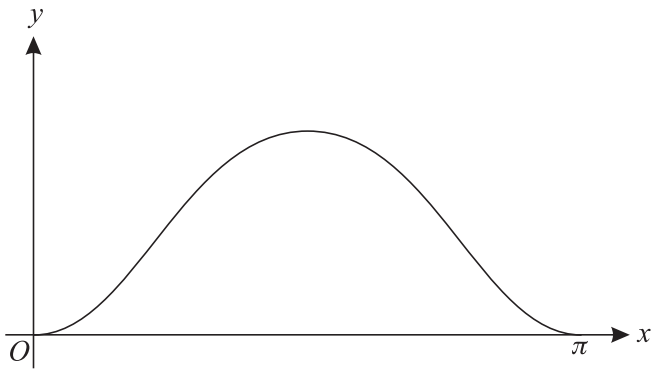
(ii) Find the equation of the tangent to the curve at the point (2, 4), giving your answer in the form $ax + by = c$. [3]

5 A curve is such that $\frac{dy}{dx} = e^{2x} - 2e^{-x}$. The point (0, 1) lies on the curve.

(i) Find the equation of the curve. [4]

(ii) The curve has one stationary point. Find the x-coordinate of this point and determine whether it is a maximum or a minimum point. [5]

6



The diagram shows the part of the curve $y = \sin^2 x$ for $0 \leqslant x \leqslant \pi$.

(i) Show that $\frac{dy}{dx} = \sin 2x$. [2]

(ii) Hence find the x-coordinates of the points on the curve at which the gradient of the curve is 0.5. [3]

(iii) By expressing $\sin^2 x$ in terms of $\cos 2x$, find the area of the region bounded by the curve and the x-axis between 0 and π. [5]