Differentiation – Parametric, Implicit, Products & Quotients

Question Paper 7

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Differentiation
Sub Topic	Differentiation – Parametric, Implicit, Products & Quotients
Booklet	Question Paper 7

Time Allowed: **68 minutes**

Score: **/56**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 **(i)** By differentiating $\frac{1}{\cos\theta}$, show that if $y = \sec\theta$ then $\frac{dy}{d\theta} = \sec\theta\tan\theta$. [3]

(ii) The parametric equations of a curve are

$$x = 1 + \tan\theta, \qquad y = \sec\theta,$$

for $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$. Show that $\frac{dy}{dx} = \sin\theta$. [3]

(iii) Find the coordinates of the point on the curve at which the gradient of the curve is $\frac{1}{2}$. [3]

2 The curve with equation $y = x^2 \ln x$, where $x > 0$, has one stationary point.

(i) Find the x-coordinate of this point, giving your answer in terms of e. [4]

(ii) Determine whether this point is a maximum or a minimum point. [2]

3 The parametric equations of a curve are

$$x = 2t + \ln t, \qquad y = t + \frac{4}{t},$$

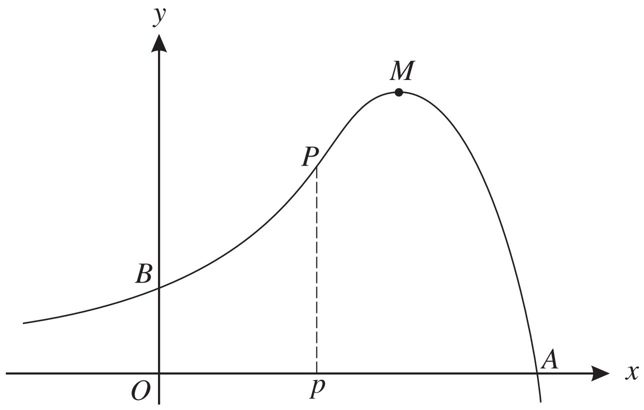
where t takes all positive values.

(i) Show that $\frac{dy}{dx} = \frac{t^2 - 4}{t(2t + 1)}$. [3]

(ii) Find the equation of the tangent to the curve at the point where $t = 1$. [3]

(iii) The curve has one stationary point. Find the y-coordinate of this point, and determine whether this point is a maximum or a minimum. [4]

4



The diagram shows the curve $y = (4 - x)e^x$ and its maximum point M. The curve cuts the x-axis at A and the y-axis at B.

(i) Write down the coordinates of A and B. [2]

(ii) Find the x-coordinate of M. [4]

(iii) The point P on the curve has x-coordinate p. The tangent to the curve at P passes through the origin O. Calculate the value of p. [5]

5 The parametric equations of a curve are

$$x = 2\theta - \sin 2\theta, \quad y = 2 - \cos 2\theta.$$

(i) Show that $\frac{dy}{dx} = \cot\theta$. [5]

(ii) Find the equation of the tangent to the curve at the point where $\theta = \frac{1}{4}\pi$. [3]

(iii) For the part of the curve where $0 < \theta < 2\pi$, find the coordinates of the points where the tangent is parallel to the x-axis. [3]

6 The equation of a curve is

$$2x^2 + 3y^2 - 2xy = 10.$$

(i) Show that $\frac{dy}{dx} = \frac{y - 2x}{3y - x}$. [4]

(ii) Find the coordinates of the points on the curve where the tangent is parallel to the x-axis. [5]