Functions

Question Paper 10

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Functions
Sub Topic	
Booklet	Question Paper 10

Time Allowed: **50 minutes**

Score: **/41**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 The function f is def ned by $f : x \mapsto 2x^2 - 12x + 7$ for $x \in \mathbb{R}$.

(i) Express $f(x)$ in the form $a(x - b)^2 - c$. [3]

(ii) State the range of f. [1]

(iii) Find the set of values of x for which $f(x) < 21$. [3]

The function g is def ned by $g : x \mapsto 2x + k$ for $x \in \mathbb{R}$.

(iv) Find the value of the constant k for which the equation $gf(x) = 0$ has two equal roots. [4]

2 The functions f and g are def ned for $x \in \mathbb{R}$ by

$$f : x \mapsto 4x - 2x^2,$$
$$g : x \mapsto 5x + 3.$$

(i) Find the range of f. [2]

(ii) Find the value of the constant k for which the equation $gf(x) = k$ has equal roots. [3]

3 Functions f and g are define by

$$f : x \mapsto k - x \quad \text{for } x \in \mathbb{R}, \text{ where } k \text{ is a constant,}$$
$$g : x \mapsto \frac{9}{x + 2} \quad \text{for } x \in \mathbb{R},\ x \neq -2.$$

(i) Find the values of k for which the equation $f(x) = g(x)$ has two equal roots and solve the equation $f(x) = g(x)$ in these cases. [6]

(ii) Solve the equation $fg(x) = 5$ when $k = 6$. [3]

(iii) Express $g^{-1}(x)$ in terms of x. [2]

4 The function f : $x \mapsto a + b\cos x$ is defined for $0 \leqslant x \leqslant 2\pi$. Given that f(0) = 10 and that $f\left(\frac{2}{3}\pi\right) = 1$, find

(i) the values of a and b, [2]

(ii) the range of f, [1]

(iii) the exact value of $f\left(\frac{5}{6}\pi\right)$. [2]

5 The function f is defined by f : $x \mapsto 2x + k$, $x \in \mathbb{R}$, where k is a constant.

(i) In the case where $k = 3$, solve the equation $ff(x) = 25$. [2]

The function g is defined by g : $x \mapsto x^2 - 6x + 8$, $x \in \mathbb{R}$.

(ii) Find the set of values of k for which the equation $f(x) = g(x)$ has no real solutions. [3]

The function h is defined by h : $x \mapsto x^2 - 6x + 8$, $x > 3$.

(iii) Find an expression for $h^{-1}(x)$. [4]