

Functions

Question Paper 3

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Functions
Sub Topic	
Booklet	Question Paper 3

Time Allowed: 56 minutes

Score: /46

Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	77.5%	70%	62.5%	57.5%	45%	<45%

1 (i) Express $2x^2 - 12x + 13$ in the form $a(x + b)^2 + c$, where a , b and c are constants. [3]

(ii) The function f is defined by $f(x) = 2x^2 - 12x + 13$ for $x \geq k$, where k is a constant. It is given that f is a one-one function. State the smallest possible value of k . [1]

The value of k is now given to be 7.

(iii) Find the range of f . [1]

(iv) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1} . [5]

2 The function f is defined by $f(x) = 4x^2 - 24x + 11$, for $x \in \mathbb{R}$.

(i) Express $f(x)$ in the form $a(x - b)^2 + c$ and hence state the coordinates of the vertex of the graph of $y = f(x)$. [4]

The function g is defined by $g(x) = 4x^2 - 24x + 11$, for $x \leq 1$.

(ii) State the range of g . [2]

(iii) Find an expression for $g^{-1}(x)$ and state the domain of g^{-1} . [4]

3 A function f is such that $f(x) = \left(\frac{x+3}{2}\right) + 1$, for $x \geq -3$. Find

(i) $f^{-1}(x)$ in the form $ax^2 + bx + c$, where a , b and c are constants, [3]

(ii) the domain of f^{-1} . [1]

4 The functions f and g are defined for $-\frac{1}{2}\pi \leq x \leq \frac{1}{2}\pi$ by

$$f(x) = \frac{1}{2}x + \frac{1}{6}\pi,$$

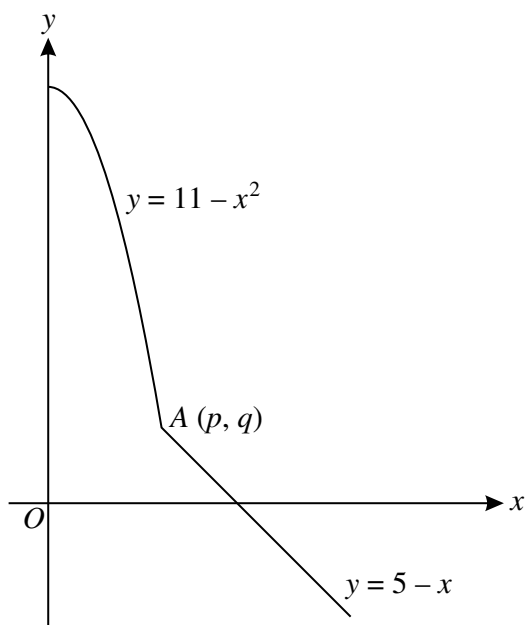
$$g(x) = \cos x.$$

Solve the following equations for $-\frac{1}{2}\pi \leq x \leq \frac{1}{2}\pi$.

(i) $gf(x) = 1$, giving your answer in terms of π . [2]

(ii) $fg(x) = 1$, giving your answers correct to 2 decimal places. [4]

5



(i) The diagram shows part of the curve $y = 11 - x^2$ and part of the straight line $y = 5 - x$ meeting at the point $A(p, q)$, where p and q are positive constants. Find the values of p and q . [3]

(ii) The function f is defined for the domain $x \geq 0$ by

$$f(x) = \begin{cases} 11 - x^2 & \text{for } 0 \leq x \leq p, \\ 5 - x & \text{for } x > p. \end{cases}$$

Express $f^{-1}(x)$ in a similar way. [5]

- 6 The function $f : x \mapsto x^2 - 4x + k$ is defined for the domain $x \geq p$, where k and p are constants.
- (i) Express $f(x)$ in the form $(x + a)^2 + b + k$, where a and b are constants. [2]
 - (ii) State the range of f in terms of k . [1]
 - (iii) State the smallest value of p for which f is one-one. [1]
 - (iv) For the value of p found in part (iii), find an expression for $f^{-1}(x)$ and state the domain of f^{-1} , giving your answers in terms of k . [4]