Functions

Question Paper 5

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Functions
Sub Topic	
Booklet	Question Paper 5

Time Allowed: **58 minutes**

Score: **/48**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 Functions f and g are defined for $x \in \mathbb{R}$ by

$$\mathrm{f} : x \mapsto 2x + 1,$$
$$\mathrm{g} : x \mapsto x^2 - 2.$$

(i) Find and simplify expressions for $\mathrm{fg}(x)$ and $\mathrm{gf}(x)$. [2]

(ii) Hence find the value of a for which $\mathrm{fg}(a) = \mathrm{gf}(a)$. [3]

(iii) Find the value of b ($b \neq a$) for which $\mathrm{g}(b) = b$. [2]

(iv) Find and simplify an expression for $\mathrm{f}^{-1}\mathrm{g}(x)$. [2]

The function h is defined by

$$\mathrm{h} : x \mapsto x^2 - 2, \quad \text{for } x \leqslant 0.$$

(v) Find an expression for $\mathrm{h}^{-1}(x)$. [2]

2 The function f is defined by $\mathrm{f} : x \mapsto \dfrac{x+3}{2x-1}$, $x \in \mathbb{R}$, $x \neq \frac{1}{2}$.

(i) Show that $\mathrm{ff}(x) = x$. [3]

(ii) Hence, or otherwise, obtain an expression for $\mathrm{f}^{-1}(x)$. [2]

3 The function f is such that $\mathrm{f}(x) = 3 - 4\cos^k x$, for $0 \leqslant x \leqslant \pi$, where k is a constant.

(i) In the case where $k = 2$,

(a) find the range of f, [2]

(b) find the exact solutions of the equation $\mathrm{f}(x) = 1$. [3]

(ii) In the case where $k = 1$,

(a) sketch the graph of $y = \mathrm{f}(x)$, [2]

(b) state, with a reason, whether f has an inverse. [1]

4 Functions f and g are defined by

$$\mathrm{f} : x \mapsto 3x - 4, \quad x \in \mathbb{R},$$
$$\mathrm{g} : x \mapsto 2(x-1)^3 + 8, \quad x > 1.$$

(i) Evaluate $\mathrm{fg}(2)$. [2]

(ii) Sketch in a single diagram the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, making clear the relationship between the graphs. [3]

(iii) Obtain an expression for $\mathrm{g}'(x)$ and use your answer to explain why g has an inverse. [3]

(iv) Express each of $\mathrm{f}^{-1}(x)$ and $\mathrm{g}^{-1}(x)$ in terms of x. [4]

5 Functions f and g are defined for $x \in \mathbb{R}$ by

$$\mathrm{f} : x \mapsto 2x + 3,$$
$$\mathrm{g} : x \mapsto x^2 - 2x.$$

Express $\mathrm{gf}(x)$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [5]

6 A function f is defined by $\mathrm{f} : x \mapsto 3 - 2\tan\left(\frac{1}{2}x\right)$ for $0 \leqslant x < \pi$.

(i) State the range of f. [1]

(ii) State the exact value of $\mathrm{f}\left(\frac{2}{3}\pi\right)$. [1]

(iii) Sketch the graph of $y = \mathrm{f}(x)$. [2]

(iv) Obtain an expression, in terms of x, for $\mathrm{f}^{-1}(x)$. [3]