Functions

Question Paper 7

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Functions
Sub Topic	
Booklet	Question Paper 7

Time Allowed: **58 minutes**

Score: **/48**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 Functions f and g are defined by

$$\mathrm{f} : x \mapsto 2x + 1, \quad x \in \mathbb{R}, \quad x > 0,$$
$$\mathrm{g} : x \mapsto \frac{2x - 1}{x + 3}, \quad x \in \mathbb{R}, \quad x \neq -3.$$

(i) Solve the equation $\mathrm{gf}(x) = x$. [3]

(ii) Express $\mathrm{f}^{-1}(x)$ and $\mathrm{g}^{-1}(x)$ in terms of x. [4]

(iii) Show that the equation $\mathrm{g}^{-1}(x) = x$ has no solutions. [3]

(iv) Sketch in a single diagram the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, making clear the relationship between the graphs. [3]

2 The function f is defined by $\mathrm{f} : x \mapsto 5 - 3\sin 2x$ for $0 \leqslant x \leqslant \pi$.

(i) Find the range of f. [2]

(ii) Sketch the graph of $y = \mathrm{f}(x)$. [3]

(iii) State, with a reason, whether f has an inverse. [1]

3 The function f is defined by $\mathrm{f} : x \mapsto 2x^2 - 12x + 13$ for $0 \leqslant x \leqslant A$, where A is a constant.

(i) Express $\mathrm{f}(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

(ii) State the value of A for which the graph of $y = \mathrm{f}(x)$ has a line of symmetry. [1]

(iii) When A has this value, find the range of f. [2]

The function g is defined by $\mathrm{g} : x \mapsto 2x^2 - 12x + 13$ for $x \geqslant 4$.

(iv) Explain why g has an inverse. [1]

(v) Obtain an expression, in terms of x, for $\mathrm{g}^{-1}(x)$. [3]

4 The function f is defined by

$$\mathrm{f} : x \mapsto 3x - 2 \quad \text{for} \quad x \in \mathbb{R}.$$

(i) Sketch, in a single diagram, the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, making clear the relationship between the two graphs. [2]

The function g is defined by

$$\mathrm{g} : x \mapsto 6x - x^2 \quad \text{for} \quad x \in \mathbb{R}.$$

(ii) Express $\mathrm{gf}(x)$ in terms of x, and hence show that the maximum value of $\mathrm{gf}(x)$ is 9. [5]

The function h is defined by

$$\mathrm{h} : x \mapsto 6x - x^2 \quad \text{for} \quad x \geqslant 3.$$

(iii) Express $6x - x^2$ in the form $a - (x - b)^2$, where a and b are positive constants. [2]

(iv) Express $\mathrm{h}^{-1}(x)$ in terms of x. [3]

5 Functions f and g are defined by

$$\mathrm{f} : x \mapsto 4x - 2k \quad \text{for } x \in \mathbb{R}, \text{ where } k \text{ is a constant,}$$

$$\mathrm{g} : x \mapsto \frac{9}{2 - x} \quad \text{for } x \in \mathbb{R}, x \neq 2.$$

(i) Find the values of k for which the equation $\mathrm{fg}(x) = x$ has two equal roots. [4]

(ii) Determine the roots of the equation $\mathrm{fg}(x) = x$ for the values of k found in part **(i)**. [3]