

Functions

Question Paper 9

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Functions
Sub Topic	
Booklet	Question Paper 9

Time Allowed: 80 minutes

Score: /66

Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	77.5%	70%	62.5%	57.5%	45%	<45%

1 The function $f : x \mapsto 2x - a$, where a is a constant, is defined for all real x .

(i) In the case where $a = 3$, solve the equation $ff(x) = 11$. [3]

The function $g : x \mapsto x^2 - 6x$ is defined for all real x .

(ii) Find the value of a for which the equation $f(x) = g(x)$ has exactly one real solution. [3]

The function $h : x \mapsto x^2 - 6x$ is defined for the domain $x \geq 3$.

(iii) Express $x^2 - 6x$ in the form $(x - p)^2 - q$, where p and q are constants. [2]

(iv) Find an expression for $h^{-1}(x)$ and state the domain of h^{-1} . [4]

2 The functions f and g are defined as follows:

$$f : x \mapsto x^2 - 2x, \quad x \in \mathbb{R},$$

$$g : x \mapsto 2x + 3, \quad x \in \mathbb{R}.$$

(i) Find the set of values of x for which $f(x) > 15$. [3]

(ii) Find the range of f and state, with a reason, whether f has an inverse. [4]

(iii) Show that the equation $gf(x) = 0$ has no real solutions. [3]

(iv) Sketch, in a single diagram, the graphs of $y = g(x)$ and $y = g^{-1}(x)$, making clear the relationship between the graphs. [2]

3 Functions f and g are defined by

$$f : x \mapsto 2x - 5, \quad x \in \mathbb{R},$$

$$g : x \mapsto \frac{4}{2-x}, \quad x \in \mathbb{R}, \quad x \neq 2.$$

(i) Find the value of x for which $fg(x) = 7$. [3]

(ii) Express each of $f^{-1}(x)$ and $g^{-1}(x)$ in terms of x . [3]

(iii) Show that the equation $f^{-1}(x) = g^{-1}(x)$ has no real roots. [3]

(iv) Sketch, on a single diagram, the graphs of $y = f(x)$ and $y = f^{-1}(x)$, making clear the relationship between these two graphs. [3]

- 4 The function f is defined by $f : x \mapsto ax + b$, for $x \in \mathbb{R}$, where a and b are constants. It is given that $f(2) = 1$ and $f(5) = 7$.

- (i) Find the values of a and b . [2]
- (ii) Solve the equation $ff(x) = 0$. [3]

- 5 The equation of a curve is $y = 8x - x^2$.

- (i) Express $8x - x^2$ in the form $a - (x + b)^2$, stating the numerical values of a and b . [3]
- (ii) Hence, or otherwise, find the coordinates of the stationary point of the curve. [2]
- (iii) Find the set of values of x for which $y \geq -20$. [3]

The function g is defined by $g : x \mapsto 8x - x^2$, for $x \geq 4$.

- (iv) State the domain and range of g^{-1} . [2]
- (v) Find an expression, in terms of x , for $g^{-1}(x)$. [3]

- 6 (i) Express $2x^2 + 8x - 10$ in the form $a(x + b)^2 + c$. [3]
- (ii) For the curve $y = 2x^2 + 8x - 10$, state the least value of y and the corresponding value of x . [2]
- (iii) Find the set of values of x for which $y \geq 14$. [3]

Given that $f : x \mapsto 2x^2 + 8x - 10$ for the domain $x \geq k$,

- (iv) find the least value of k for which f is one-one, [1]
- (v) express $f^{-1}(x)$ in terms of x in this case. [3]