

Integration – Trig, Log & Exponential Functions

Question Paper 1

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Integration
Sub Topic	Integration – Trig, Log & Exponential Functions
Booklet	Question Paper 1

Time Allowed: 56 minutes

Score: /46

Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	77.5%	70%	62.5%	57.5%	45%	<45%

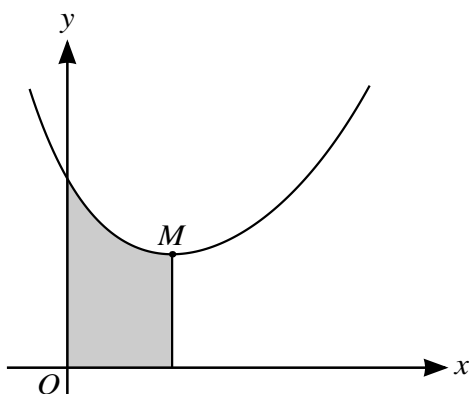
1 (i) Prove that $2 \operatorname{cosec} 2\theta \tan \theta \equiv \sec^2 \theta$. [3]

(ii) Hence

(a) solve the equation $2 \operatorname{cosec} 2\theta \tan \theta = 5$ for $0 < \theta < \pi$, [3]

(b) find the exact value of $\int_0^{\frac{1}{6}\pi} 2 \operatorname{cosec} 4x \tan 2x \, dx$. [4]

2

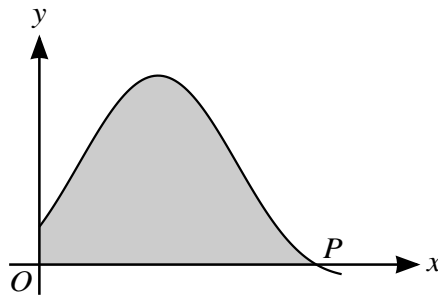


The diagram shows the curve $y = e^x + 4e^{-2x}$ and its minimum point M .

(i) Show that the x -coordinate of M is $\ln 2$. [3]

(ii) The region shaded in the diagram is enclosed by the curve and the lines $x = 0$, $x = \ln 2$ and $y = 0$. Use integration to show that the area of the shaded region is $\frac{5}{2}$. [4]

3



The diagram shows part of the curve with equation

$$y = 4 \sin^2 x + 8 \sin x + 3$$

and its point of intersection P with the x -axis.

(i) Find the exact x -coordinate of P . [3]

(ii) Show that the equation of the curve can be written

$$y = 5 + 8 \sin x - 2 \cos 2x,$$

and use integration to find the exact area of the shaded region enclosed by the curve and the axes. [6]

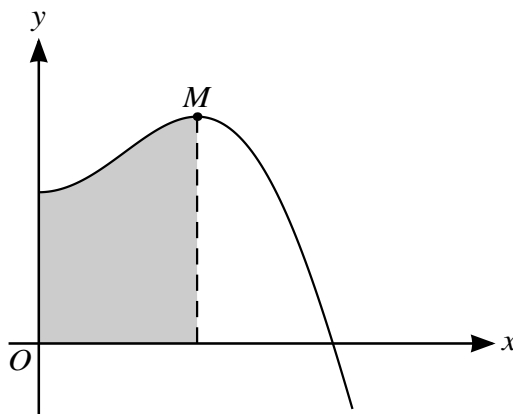
4 (a) Find $\int 4 \cos^2\left(\frac{1}{2}\theta\right) d\theta$. [3]

(b) Find the exact value of $\int_0^6 \frac{1}{2x+3} dx$. [4]

5 (i) Find $\int_0^a (e^{-x} + 6e^{-3x}) dx$, where a is a positive constant. [4]

(ii) Deduce the value of $\int_0^\infty (e^{-x} + 6e^{-3x}) dx$. [1]

6



The diagram shows part of the curve

$$y = 2 \cos x - \cos 2x$$

and its maximum point M . The shaded region is bounded by the curve, the axes and the line through M parallel to the y-axis.

(i) Find the exact value of the x -coordinate of M . [4]

(ii) Find the exact value of the area of the shaded region. [4]