

Integration – Trig, Log & Exponential Functions

Question Paper 3

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Integration
Sub Topic	Integration – Trig, Log & Exponential Functions
Booklet	Question Paper 3

Time Allowed: 59 minutes

Score: /49

Percentage: /100

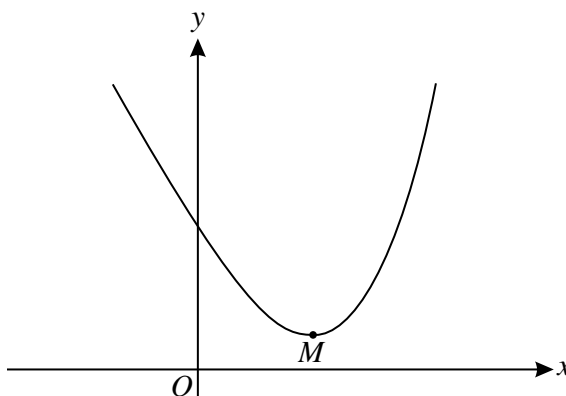
Grade Boundaries:

A*	A	B	C	D	E	U
>85%	77.5%	70%	62.5%	57.5%	45%	<45%

1 (i) Show that $(2 \sin x + \cos x)^2$ can be written in the form $\frac{5}{2} + 2 \sin 2x - \frac{3}{2} \cos 2x$. [5]

(ii) Hence find the exact value of $\int_0^{\frac{1}{4}\pi} (2 \sin x + \cos x)^2 dx$. [4]

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The diagram shows the curve $y = 4e^{\frac{1}{2}x} - 6x + 3$ and its minimum point M .

(i) Show that the x -coordinate of M can be written in the form $\ln a$, where the value of a is to be stated. [5]

(ii) Find the exact value of the area of the region enclosed by the curve and the lines $x = 0$, $x = 2$ and $y = 0$. [4]

3 Show that $\int_2^6 \frac{2}{4x+1} dx = \ln \frac{5}{3}$. [5]

4 Find the exact value of the positive constant k for which

$$\int_0^k e^{4x} dx = \int_0^{2k} e^x dx. \quad [6]$$

5 (i) Express $\cos^2 x$ in terms of $\cos 2x$. [1]

(ii) Hence show that

$$\int_0^{\frac{1}{6}\pi} (\cos^2 x + \sin 2x) dx = \frac{1}{8}\sqrt{3} + \frac{1}{12}\pi + \frac{1}{4}. \quad [5]$$

6 (a) Find $\int 4e^x(3 + e^{2x}) dx$. [4]

(b) Show that $\int_{-\frac{1}{4}\pi}^{\frac{1}{4}\pi} (3 + 2 \tan^2 \theta) d\theta = \frac{1}{2}(8 + \pi)$. [4]

7 (a) Find the value of $\int_0^{\frac{2}{3}\pi} \sin\left(\frac{1}{2}x\right) dx$. [3]

(b) Find $\int e^{-x}(1 + e^x) dx$. [3]