

Integration – Trig, Log & Exponential Functions

Question Paper 4

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Integration
Sub Topic	Integration – Trig, Log & Exponential Functions
Booklet	Question Paper 4

Time Allowed: 56 minutes

Score: /46

Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	77.5%	70%	62.5%	57.5%	45%	<45%

1 Show that $\int_0^1 (e^x + 1)^2 dx = \frac{1}{2}e^2 + 2e - \frac{3}{2}$. [5]

2 (a) Find $\int e^{1-2x} dx$. [2]

(b) Express $\sin^2 3x$ in terms of $\cos 6x$ and hence find $\int \sin^2 3x dx$. [4]

3 Show that $\int_0^6 \frac{1}{x+2} dx = 2 \ln 2$. [4]

4 (i) By differentiating $\frac{\cos x}{\sin x}$, show that if $y = \cot x$ then $\frac{dy}{dx} = -\operatorname{cosec}^2 x$. [3]

(ii) By expressing $\cot^2 x$ in terms of $\operatorname{cosec}^2 x$ and using the result of part (i), show that

$$\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} \cot^2 x dx = 1 - \frac{1}{4}\pi. \quad [4]$$

(iii) Express $\cos 2x$ in terms of $\sin^2 x$ and hence show that $\frac{1}{1 - \cos 2x}$ can be expressed as $\frac{1}{2} \operatorname{cosec}^2 x$.
Hence, using the result of part (i), find

$$\int \frac{1}{1 - \cos 2x} dx. \quad [3]$$

5 (a) Show that $\int_0^{\frac{1}{4}\pi} \cos 2x dx = \frac{1}{2}$. [2]

(b) By using an appropriate trigonometrical identity, find the exact value of

$$\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} 3 \tan^2 x dx. \quad [4]$$

6 (i) Express $\cos^2 2x$ in terms of $\cos 4x$. [2]

(ii) Hence find the exact value of $\int_0^{\frac{1}{8}\pi} \cos^2 2x \, dx$. [4]

7 (a) Find the exact value of $\int_0^{\frac{1}{3}\pi} (\sin 2x + \sec^2 x) \, dx$. [5]

(b) Show that $\int_1^4 \left(\frac{1}{2x} + \frac{1}{x+1} \right) dx = \ln 5$. [4]