

# Integration – Trig, Log & Exponential Functions

## Question Paper 5

<b>Level</b>	International A Level
<b>Subject</b>	Maths
<b>Exam Board</b>	CIE
<b>Topic</b>	Integration
<b>Sub Topic</b>	Integration – Trig, Log & Exponential Functions
<b>Booklet</b>	Question Paper 5

**Time Allowed:** 60 minutes

**Score:** /50

**Percentage:** /100

**Grade Boundaries:**

A*	A	B	C	D	E	U
>85%	77.5%	70%	62.5%	57.5%	45%	<45%

1 (a) Find the equation of the tangent to the curve  $y = \ln(3x - 2)$  at the point where  $x = 1$ . [4]

(b) (i) Find the value of the constant  $A$  such that

$$\frac{6x}{3x-2} \equiv 2 + \frac{A}{3x-2}. \quad [2]$$

(ii) Hence show that  $\int_2^6 \frac{6x}{3x-2} dx = 8 + \frac{8}{3} \ln 2$ . [5]

2 Show that  $\int_1^2 \left( \frac{1}{x} - \frac{4}{2x+1} \right) dx = \ln \frac{18}{25}$ . [6]

3 (i) (a) Prove the identity

$$\sec^2 x + \sec x \tan x \equiv \frac{1 + \sin x}{\cos^2 x}.$$

(b) Hence prove that

$$\sec^2 x + \sec x \tan x \equiv \frac{1}{1 - \sin x}. \quad [3]$$

(ii) By differentiating  $\frac{1}{\cos x}$ , show that if  $y = \sec x$  then  $\frac{dy}{dx} = \sec x \tan x$ . [3]

(iii) Using the results of parts (i) and (ii), find the exact value of

$$\int_0^{\frac{1}{4}\pi} \frac{1}{1 - \sin x} dx. \quad [3]$$

4 Find the exact value of  $\int_0^{\frac{1}{6}\pi} (\cos 2x + \sin x) dx$ . [5]

5 Show that

$$\int_1^4 \frac{1}{2x+1} dx = \frac{1}{2} \ln 3. \quad [4]$$

6 (i) Prove the identity

$$(\cos x + 3 \sin x)^2 \equiv 5 - 4 \cos 2x + 3 \sin 2x. \quad [4]$$

(ii) Using the identity, or otherwise, find the exact value of

$$\int_0^{\frac{1}{4}\pi} (\cos x + 3 \sin x)^2 dx. \quad [4]$$

7 (i) Express  $\cos^2 x$  in terms of  $\cos 2x$ . [1]

(ii) Hence show that

$$\int_0^{\frac{1}{3}\pi} \cos^2 x dx = \frac{1}{6}\pi + \frac{1}{8}\sqrt{3}. \quad [4]$$

(iii) By using an appropriate trigonometrical identity, deduce the exact value of

$$\int_0^{\frac{1}{3}\pi} \sin^2 x dx. \quad [3]$$