Integration – Trig, Log & Exponential Functions

Question Paper 6

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Integration
Sub Topic	Integration – Trig, Log & Exponential Functions
Booklet	Question Paper 6

Time Allowed: **58 minutes**

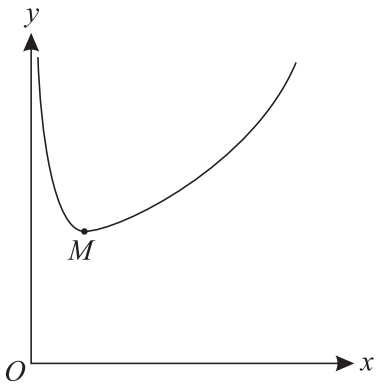
Score: **/48**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1



The diagram shows the part of the curve $y = \dfrac{e^{2x}}{x}$ for $x > 0$, and its minimum point M.

(i) Find the coordinates of M. [5]

(ii) Use the trapezium rule with 2 intervals to estimate the value of

$$\int_1^2 \frac{e^{2x}}{x}\,dx,$$

giving your answer correct to 1 decimal place. [3]

(iii) State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part **(ii)**. [1]

2 **(i)** Given that $y = \tan 2x$, find $\dfrac{dy}{dx}$. [2]

(ii) Hence, or otherwise, show that

$$\int_0^{\frac{1}{6}\pi} \sec^2 2x\,dx = \tfrac{1}{2}\sqrt{3},$$

and, by using an appropriate trigonometrical identity, find the exact value of $\displaystyle\int_0^{\frac{1}{6}\pi} \tan^2 2x\,dx$. [6]

(iii) Use the identity $\cos 4x \equiv 2\cos^2 2x - 1$ to find the exact value of

$$\int_0^{\frac{1}{6}\pi} \frac{1}{1+\cos 4x}\,dx.$$ [2]

3 **(i)** Differentiate $\ln(2x + 3)$. [2]

(ii) Hence, or otherwise, show that

$$\int_{-1}^{3} \frac{1}{2x+3}\,\mathrm{d}x = \ln 3.$$ [3]

(iii) Find the quotient and remainder when $4x^2 + 8x$ is divided by $2x + 3$. [3]

(iv) Hence show that

$$\int_{-1}^{3} \frac{4x^2+8x}{2x+3}\,\mathrm{d}x = 12 - 3\ln 3.$$ [3]

4 **(i)** By sketching a suitable pair of graphs, show that there is only one value of x that is a root of the equation

$$\frac{1}{x} = \ln x.$$ [2]

(ii) Verify by calculation that this root lies between 1 and 2. [2]

(iii) Show that this root also satisfies the equation

$$x = \mathrm{e}^{\frac{1}{x}}.$$ [1]

(iv) Use the iterative formula

$$x_{n+1} = \mathrm{e}^{\frac{1}{x_n}},$$

with initial value $x_1 = 1.8$, to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

5 **(i)** By expanding $\sin(2x + x)$ and using double-angle formulae, show that

$$\sin 3x = 3\sin x - 4\sin^3 x.$$ [5]

(ii) Hence show that

$$\int_0^{\frac{1}{3}\pi} \sin^3 x \, \mathrm{d}x = \frac{5}{24}.$$ [5]