Integration – Trig, Log & Exponential Functions

Question Paper 7

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Integration
Sub Topic	Integration – Trig, Log & Exponential Functions
Booklet	Question Paper 7

Time Allowed: **74 minutes**

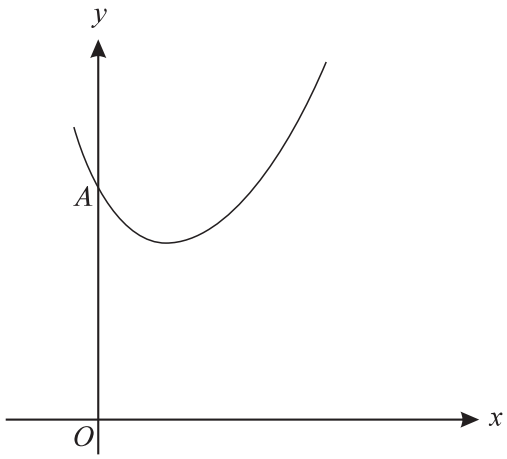
Score: **/61**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1



The diagram shows the curve $y = 2e^{x} + 3e^{-2x}$. The curve cuts the y-axis at A.

(i) Write down the coordinates of A. [1]

(ii) Find the equation of the tangent to the curve at A, and state the coordinates of the point where this tangent meets the x-axis. [6]

(iii) Calculate the area of the region bounded by the curve and by the lines $x = 0$, $y = 0$ and $x = 1$, giving your answer correct to 2 significant figures. [4]

2 **(i)** Express $\cos\theta + \sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$, giving the exact values of R and α. [3]

(ii) Hence show that

$$\frac{1}{(\cos\theta + \sin\theta)^2} = \tfrac{1}{2}\sec^2(\theta - \tfrac{1}{4}\pi).$$ [1]

(iii) By differentiating $\dfrac{\sin x}{\cos x}$, show that if $y = \tan x$ then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \sec^2 x$. [3]

(iv) Using the results of parts **(ii)** and **(iii)**, show that

$$\int_0^{\frac{1}{2}\pi} \frac{1}{(\cos\theta + \sin\theta)^2}\,\mathrm{d}\theta = 1.$$ [3]

3 **(i)** By expanding $\cos(2x + x)$, show that

$$\cos 3x \equiv 4\cos^3 x - 3\cos x.$$ [5]

(ii) Hence, or otherwise, show that

$$\int_0^{\frac{1}{2}\pi} \cos^3 x \, dx = \frac{2}{3}.$$ [5]

4 **(i)** By differentiating $\frac{\cos x}{\sin x}$, show that if $y = \cot x$ then $\frac{dy}{dx} = -\operatorname{cosec}^2 x$. [3]

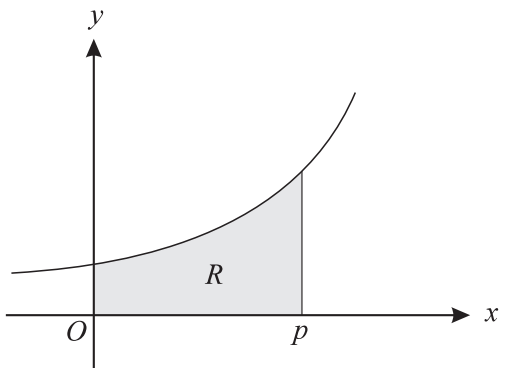
(ii) Hence show that $\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} \operatorname{cosec}^2 x \, dx = \sqrt{3}$. [2]

By using appropriate trigonometrical identities, find the exact value of

(iii) $\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} \cot^2 x \, dx$, [3]

(iv) $\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} \frac{1}{1 - \cos 2x} \, dx$. [3]

5



The diagram shows the curve $y = e^{2x}$. The shaded region R is bounded by the curve and by the lines $x = 0$, $y = 0$ and $x = p$.

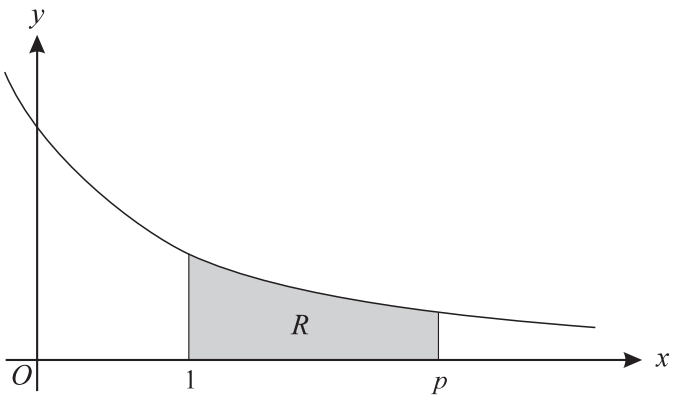
(i) Find, in terms of p, the area of R. [3]

(ii) Hence calculate the value of p for which the area of R is equal to 5. Give your answer correct to 2 significant figures. [3]

6 Solve the inequality $|2x - 1| < |3x|$. [4]

7 **(a)** Find the value of $\int_0^{\frac{1}{2}\pi} (\sin 2x + \cos x)\,\mathrm{d}x$. [4]

(b)



The diagram shows part of the curve $y = \dfrac{1}{x+1}$. The shaded region R is bounded by the curve and by the lines $x = 1$, $y = 0$ and $x = p$.

(i) Find, in terms of p, the area of R. [3]

(ii) Hence find, correct to 1 decimal place, the value of p for which the area of R is equal to 2. [2]