Integration – Trapezium Rule

Question Paper 2

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Integration
Sub Topic	Integration – Trapezium Rule
Booklet	Question Paper 2

Time Allowed: **80 minutes**

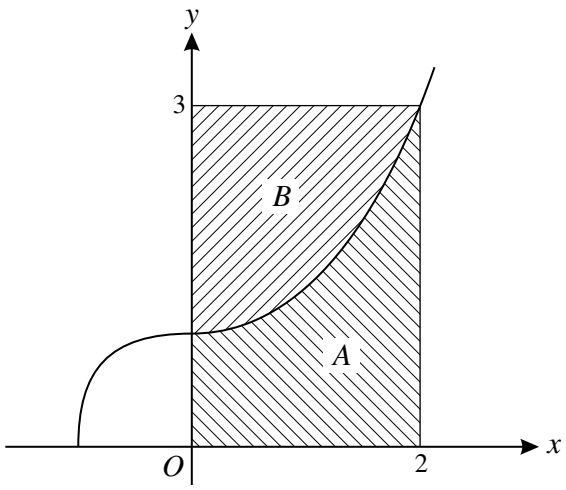
Score: **/66**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1

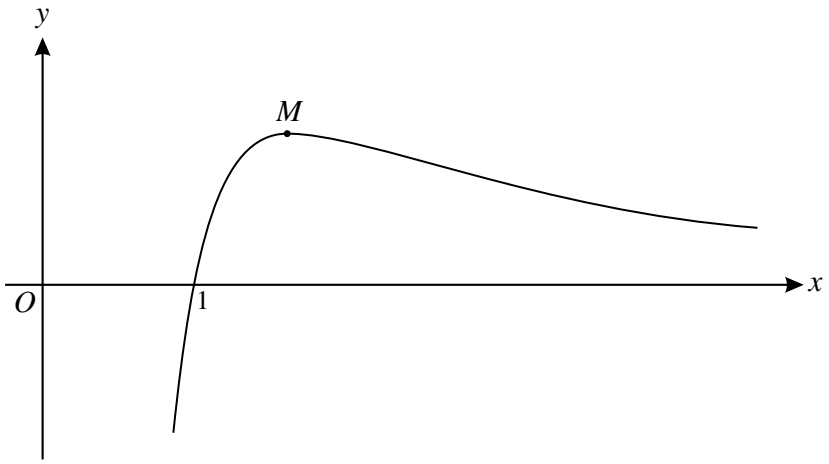


The diagram shows the curve $y = \sqrt{(1 + x^3)}$. Region A is bounded by the curve and the lines $x = 0$, $x = 2$ and $y = 0$. Region B is bounded by the curve and the lines $x = 0$ and $y = 3$.

(i) Use the trapezium rule with two intervals to find an approximation to the area of region A. Give your answer correct to 2 decimal places. [3]

(ii) Deduce an approximation to the area of region B and explain why this approximation underestimates the true area of region B. [2]

2



The diagram shows the curve $y = \dfrac{\ln x}{x^2}$ and its maximum point M.

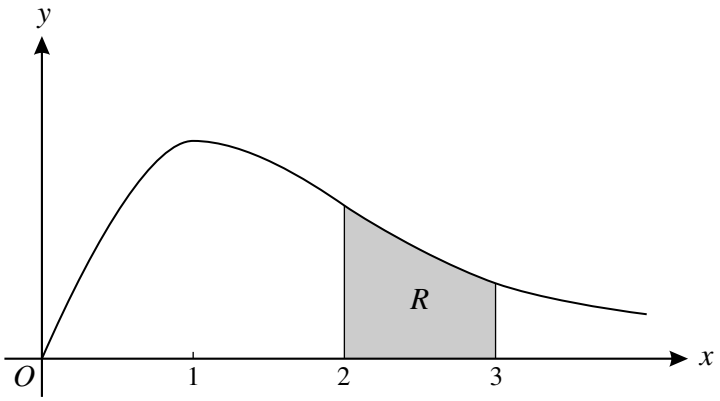
(i) Find the exact coordinates of M. [5]

(ii) Use the trapezium rule with three intervals to estimate the value of

$$\int_1^4 \frac{\ln x}{x^2}\,\mathrm{d}x,$$

giving your answer correct to 2 decimal places. [3]

3

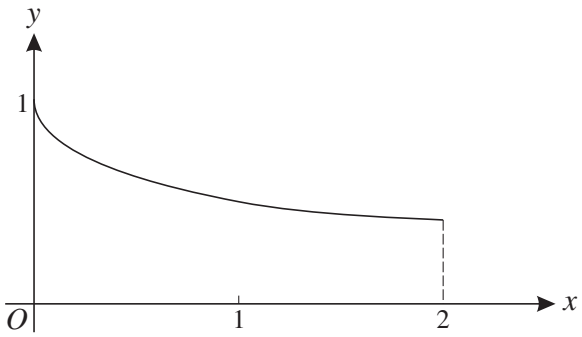


The diagram shows part of the curve $y = xe^{-x}$. The shaded region R is bounded by the curve and by the lines $x = 2$, $x = 3$ and $y = 0$.

(i) Use the trapezium rule with two intervals to estimate the area of R, giving your answer correct to 2 decimal places. [3]

(ii) State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the area of R. [1]

4



The diagram shows the curve $y = \dfrac{1}{1+\sqrt{x}}$ for values of x from 0 to 2.

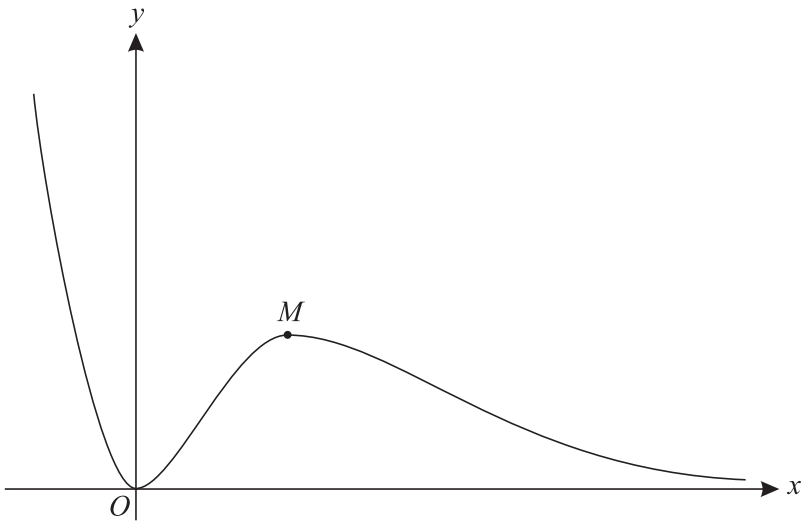
(i) Use the trapezium rule with two intervals to estimate the value of

$$\int_0^2 \frac{1}{1+\sqrt{x}}\,\mathrm{d}x,$$

giving your answer correct to 2 decimal places. [3]

(ii) State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part **(i)**. [1]

5



The diagram shows the curve $y = x^2e^{-x}$ and its maximum point M.

(i) Find the x-coordinate of M. [4]

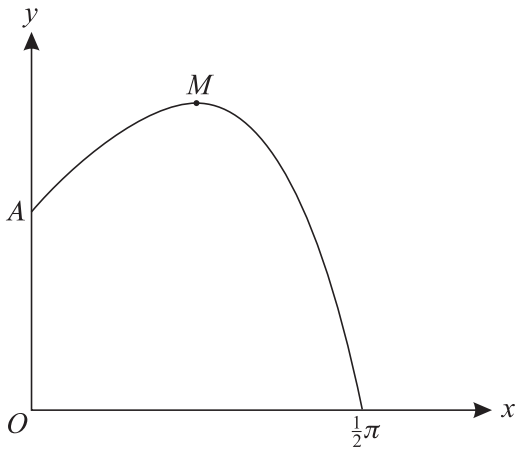
(ii) Show that the tangent to the curve at the point where $x = 1$ passes through the origin. [3]

(iii) Use the trapezium rule, with two intervals, to estimate the value of

$$\int_1^3 x^2e^{-x}\,dx,$$

giving your answer correct to 2 decimal places. [3]

6



The diagram shows the part of the curve $y = e^x \cos x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$. The curve meets the y-axis at the point A. The point M is a maximum point.

(i) Write down the coordinates of A. [1]

(ii) Find the x-coordinate of M. [4]

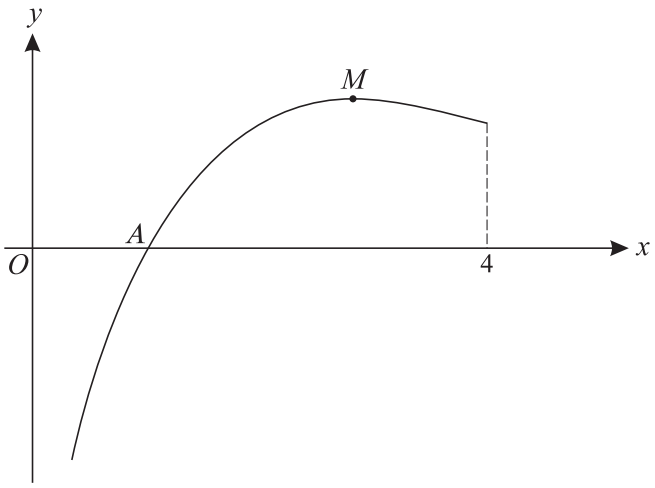
(iii) Use the trapezium rule with three intervals to estimate the value of

$$\int_0^{\frac{1}{2}\pi} e^x \cos x \, dx,$$

giving your answer correct to 2 decimal places. [3]

(iv) State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part **(iii)**. [1]

7



The diagram shows the part of the curve $y = \dfrac{\ln x}{x}$ for $0 < x \leqslant 4$. The curve cuts the x-axis at A and its maximum point is M.

(i) Write down the coordinates of A. [1]

(ii) Show that the x-coordinate of M is e, and write down the y-coordinate of M in terms of e. [5]

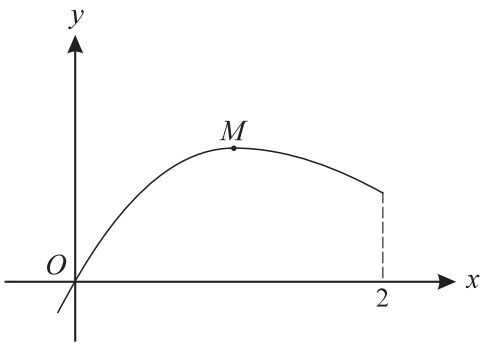
(iii) Use the trapezium rule with three intervals to estimate the value of

$$\int_1^4 \frac{\ln x}{x}\,\mathrm{d}x,$$

correct to 2 decimal places. [3]

(iv) State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part **(iii)**. [1]

8



The diagram shows the part of the curve $y = xe^{-x}$ for $0 \leqslant x \leqslant 2$, and its maximum point M.

(i) Find the x-coordinate of M. [4]

(ii) Use the trapezium rule with two intervals to estimate the value of

$$\int_0^2 xe^{-x}\,dx,$$

giving your answer correct to 2 decimal places. [3]

(iii) State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part **(ii)**. [1]

9 The equation of a curve is $y = \dfrac{}{1 + \tan x}$.

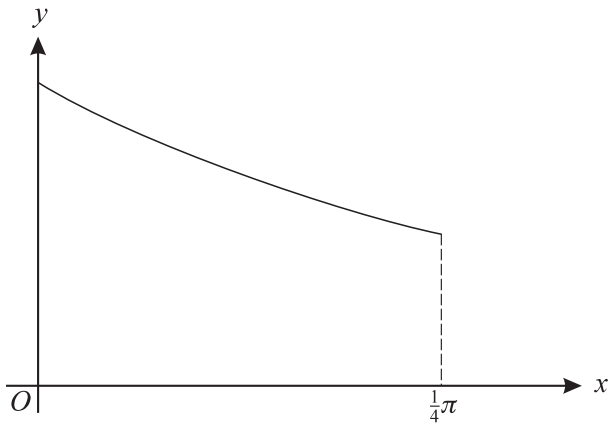
(i) Show, by differentiation, that the gradient of the curve is always negative. [4]

(ii) Use the trapezium rule with 2 intervals to estimate the value of

$$\int_0^{\frac{1}{4}\pi} \frac{1}{1 + \tan x}\,\mathrm{d}x,$$

giving your answer correct to 2 significant figures. [3]

(iii)



The diagram shows a sketch of the curve for $0 \leqslant x \leqslant \frac{1}{4}\pi$. State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part **(ii)**. [1]