

Numerical Solutions of Equations

Question Paper 1

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Numerical Solutions of Equations
Sub Topic	
Booklet	Question Paper 1

Time Allowed: 58 minutes

Score: /48

Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

- 1 (i) Given that $\int_0^a (3e^{\frac{1}{2}x} + 1) dx = 10$, show that the positive constant a satisfies the equation

$$a = 2 \ln \left(\frac{16 - a}{6} \right). \quad [5]$$

- (ii) Use the iterative formula $a_{n+1} = 2 \ln \left(\frac{16 - a_n}{6} \right)$ with $a_1 = 2$ to find the value of a correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

- 2 (i) By sketching a suitable pair of graphs, show that the equation

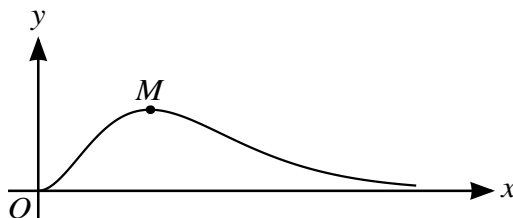
$$|3x| = 16 - x^4$$

has two real roots. [3]

- (ii) Use the iterative formula $x_{n+1} = \sqrt[4]{16 - 3x_n}$ to find one of the real roots correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

- (iii) Hence find the coordinates of each of the points of intersection of the graphs $y = |3x|$ and $y = 16 - x^4$, giving your answers correct to 3 decimal places. [2]

3



The diagram shows part of the curve $y = \frac{x^2}{1 + e^{3x}}$ and its maximum point M . The x -coordinate of M is denoted by m .

- (i) Find $\frac{dy}{dx}$ and hence show that m satisfies the equation $x = \frac{2}{3}(1 + e^{-3x})$. [4]
- (ii) Show by calculation that m lies between 0.7 and 0.8. [2]
- (iii) Use an iterative formula based on the equation in part (i) to find m correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

- 4 (i) By sketching a suitable pair of graphs, show that the equation

$$3 \ln x = 15 - x^3$$

has exactly one real root. [3]

- (ii) Show by calculation that the root lies between 2.0 and 2.5. [2]

- (iii) Use the iterative formula $x_{n+1} = \sqrt[3]{(15 - 3 \ln x_n)}$ to find the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

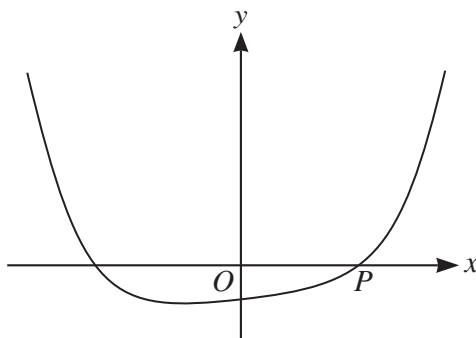
- 5 It is given that $\int_0^a (\frac{1}{2}e^{3x} + x^2) dx = 10$, where a is a positive constant.

- (i) Show that $a = \frac{1}{3} \ln(61 - 2a^3)$. [4]

- (ii) Show by calculation that the value of a lies between 1.0 and 1.5. [2]

- (iii) Use an iterative formula, based on the equation in part (i), to find the value of a correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

6



The diagram shows the curve $y = x^4 + 2x - 9$. The curve cuts the positive x -axis at the point P .

- (i) Verify by calculation that the x -coordinate of P lies between 1.5 and 1.6. [2]

- (ii) Show that the x -coordinate of P satisfies the equation

$$x = \sqrt[3]{\left(\frac{9}{x} - 2\right)}. \quad [1]$$

- (iii) Use the iterative formula

$$x_{n+1} = \sqrt[3]{\left(\frac{9}{x_n} - 2\right)}$$

to determine the x -coordinate of P correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]