Arithmetic & Geometric series

Question Paper 1

Level	A Level
Subject	Maths
Exam Board	OCR - MEI
Module	Core 2
Topic	Sequences & Series
Sub Topic	Arithmetic & Geometric series
Booklet	Question Paper 1

Time Allowed: **53 minutes**

Score: **/44**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	77.5%	70%	62.5%	57.5%	45%	<45%

1 An arithmetic progression has tenth term 11.1 and fiftieth term 7.1. Find the first term and the common difference. Find also the sum of the first fifty terms of the progression. **[5]**

2 Jill has 3 daughters and no sons. They are generation 1 of Jill's descendants.

Each of her daughters has 3 daughters and no sons. Jill's 9 granddaughters are generation 2 of her descendants. Each of her granddaughters has 3 daughters and no sons; they are descendant generation 3.

Jill decides to investigate what would happen if this pattern continues, with each descendant having 3 daughters and no sons.

(i) How many of Jill's descendants would there be in generation 8? **[2]**

(ii) How many of Jill's descendants would there be altogether in the first 15 generations? **[3]**

(iii) After n generations, Jill would have over a million descendants altogether. Show that n satisfies the inequality

$$n > \frac{\log_{10} 2000003}{\log_{10} 3} - 1.$$

Hence find the least possible value of n. **[4]**

(iv) How many **fewer** descendants would Jill have altogether in 15 generations if instead of having 3 daughters, she and each subsequent descendant has 2 daughters? **[3]**

3 **(i)** Find $\sum_{r=1}^{5} \frac{21}{r+2}$. **[2]**

(ii) A sequence is defined by

$$u_1 = a, \text{ where } a \text{ is an unknown constant,}$$
$$u_{n+1} = u_n + 5.$$

Find, in terms of a, the tenth term and the sum of the first ten terms of this sequence. **[3]**

4 The second term of a geometric progression is 24. The sum to infinity of this progression is 150. Write down two equations in a and r, where a is the first term and r is the common ratio. Solve your equations to find the possible values of a and r. **[5]**

5 S is the sum to infinity of a geometric progression with first term a and common ratio r.

(i) Another geometric progression has first term $2a$ and common ratio r. Express the sum to infinity of this progression in terms of S. **[1]**

(ii) A third geometric progression has first term a and common ratio r^2. Express, in its simplest form, the sum to infinity of this progression in terms of S and r. **[2]**

6 Find the second and third terms in the sequence given by

$$u_1 = 5,$$
$$u_{n+1} = u_n + 3.$$

Find also the sum of the first 50 terms of this sequence. **[4]**

7 A geometric progression has first term a and common ratio r. The second term is 6 and the sum to infinity is 25.

(i) Write down two equations in a and r. Show that one possible value of a is 10 and find the other possible value of a. Write down the corresponding values of r. **[7]**

(ii) Show that the ratio of the nth terms of the two geometric progressions found in part **(i)** can be written as $2^{n-2} : 3^{n-2}$. **[3]**