Numerical Solutions of Equations

Question Paper 2

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Numerical Solutions of Equations
Sub Topic	
Booklet	Question Paper 2

Time Allowed: **51 minutes**

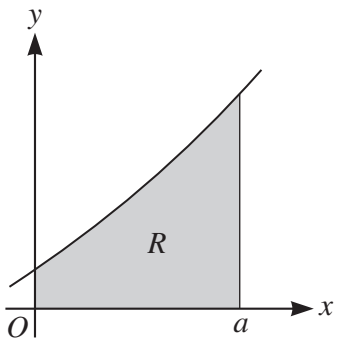
Score: **/42**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1



The diagram shows part of the curve $y = 8x + \frac{1}{2}e^x$. The shaded region R is bounded by the curve and by the lines $x = 0$, $y = 0$ and $x = a$, where a is positive. The area of R is equal to $\frac{1}{2}$.

(i) Find an equation satisfied by a, and show that the equation can be written in the form

$$a = \sqrt{\left(\frac{2 - e^a}{8}\right)}.$$ [5]

(ii) Verify by calculation that the equation $a = \sqrt{\left(\frac{2 - e^a}{8}\right)}$ has a root between 0.2 and 0.3. [2]

(iii) Use the iterative formula $a_{n+1} = \sqrt{\left(\frac{2 - e^{a_n}}{8}\right)}$ to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

2 **(i)** By sketching a suitable pair of graphs, show that the equation

$$\cot x = 4x - 2,$$

where x is in radians, has only one root for $0 \leqslant x \leqslant \frac{1}{2}\pi$. [2]

(ii) Verify by calculation that this root lies between $x = 0.7$ and $x = 0.9$. [2]

(iii) Show that this root also satisfies the equation

$$x = \frac{1 + 2\tan x}{4\tan x}.$$ [1]

(iv) Use the iterative formula $x_{n+1} = \frac{1 + 2\tan x_n}{4\tan x_n}$ to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

3 **(i)** By sketching a suitable pair of graphs, show that the equation

$$3e^x = 8 - 2x$$

has only one root. [2]

(ii) Verify by calculation that this root lies between $x = 0.7$ and $x = 0.8$. [2]

(iii) Show that this root also satisfies the equation

$$x = \ln\left(\frac{8 - 2x}{3}\right).$$ [1]

(iv) Use the iterative formula $x_{n+1} = \ln\left(\frac{8 - 2x_n}{3}\right)$ to determine this root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

4 A curve has parametric equations

$$x = \frac{1}{(2t + 1)^2}, \quad y = \sqrt{(t + 2)}.$$

The point P on the curve has parameter p and it is given that the gradient of the curve at P is -1.

(i) Show that $p = (p + 2)^{\frac{1}{6}} - \frac{1}{2}$. [6]

(ii) Use an iterative process based on the equation in part **(i)** to find the value of p correct to 3 decimal places. Use a starting value of 0.7 and show the result of each iteration to 5 decimal places. [3]

5 **(i)** Verify by calculation that the cubic equation

$$x^3 - 2x^2 + 5x - 3 = 0$$

has a root that lies between $x = 0.7$ and $x = 0.8$. [2]

(ii) Show that this root also satisfies an equation of the form

$$x = \frac{ax^2 + 3}{x^2 + b},$$

where the values of a and b are to be found. [2]

(iii) With these values of a and b, use the iterative formula

$$x_{n+1} = \frac{ax_n^2 + 3}{x_n^2 + b}$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]