

Numerical Solutions of Equations

Question Paper 3

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Numerical Solutions of Equations
Sub Topic	
Booklet	Question Paper 3

Time Allowed: 53 minutes

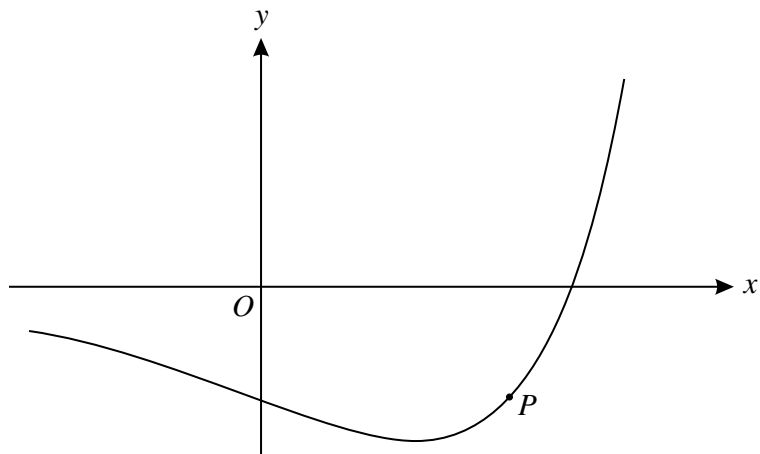
Score: /44

Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

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The diagram shows the curve $y = (x - 4)e^{\frac{1}{2}x}$. The curve has a gradient of 3 at the point P .

- (i) Show that the x -coordinate of P satisfies the equation

$$x = 2 + 6e^{-\frac{1}{2}x}. \quad [4]$$

- (ii) Verify that the equation in part (i) has a root between $x = 3.1$ and $x = 3.3$. [2]

- (iii) Use the iterative formula $x_{n+1} = 2 + 6e^{-\frac{1}{2}x_n}$ to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

- 2 (i) By sketching a suitable pair of graphs, show that the equation

$$e^{2x} = 14 - x^2$$

has exactly two real roots. [3]

- (ii) Show by calculation that the positive root lies between 1.2 and 1.3. [2]

- (iii) Show that this root also satisfies the equation

$$x = \frac{1}{2} \ln(14 - x^2). \quad [1]$$

- (iv) Use an iteration process based on the equation in part (iii), with a suitable starting value, to find the root correct to 2 decimal places. Give the result of each step of the process to 4 decimal places. [3]

- 3 The sequence x_1, x_2, x_3, \dots defined by

$$x_1 = 1, \quad x_{n+1} = \frac{1}{2}\sqrt[3]{(x_n^2 + 6)}$$

converges to the value α .

- (i) Find the value of α correct to 3 decimal places. Show your working, giving each calculated value of the sequence to 5 decimal places. [3]
- (ii) Find, in the form $ax^3 + bx^2 + c = 0$, an equation of which α is a root. [2]

- 4 The curve with equation $y = \frac{6}{x^2}$ intersects the line $y = x + 1$ at the point P .

- (i) Verify by calculation that the x -coordinate of P lies between 1.4 and 1.6. [2]
- (ii) Show that the x -coordinate of P satisfies the equation

$$x = \sqrt{\left(\frac{6}{x+1}\right)}. \quad [2]$$

- (iii) Use the iterative formula

$$x_{n+1} = \sqrt{\left(\frac{6}{x_n + 1}\right)},$$

with initial value $x_1 = 1.5$, to determine the x -coordinate of P correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

- 5 The sequence of values given by the iterative formula

$$x_{n+1} = \frac{7x_n}{8} + \frac{5}{2x_n^4},$$

with initial value $x_1 = 1.7$, converges to α .

- (i) Use this iterative formula to determine α correct to 2 decimal places, giving the result of each iteration to 4 decimal places. [3]
- (ii) State an equation that is satisfied by α and hence show that $\alpha = \sqrt[5]{20}$. [2]

6 The equation of a curve is

$$x^2 + 2xy - y^2 + 8 = 0.$$

- (i) Show that the tangent to the curve at the point $(-2, 2)$ is parallel to the x -axis. [4]
- (ii) Find the equation of the tangent to the curve at the other point on the curve for which $x = -2$, giving your answer in the form $y = mx + c$. [5]