Numerical Solutions of Equations

Question Paper 4

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Numerical Solutions of Equations
Sub Topic	
Booklet	Question Paper 4

Time Allowed: **60 minutes**

Score: **/50**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 **(i)** By sketching a suitable pair of graphs, show that the equation

$$e^{2x} = 2 - x$$

has only one root. [2]

(ii) Verify by calculation that this root lies between $x = 0$ and $x = 0.5$. [2]

(iii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \tfrac{1}{2}\ln(2 - x_n)$$

converges, then it converges to the root of the equation in part **(i)**. [1]

(iv) Use this iterative formula, with initial value $x_1 = 0.25$, to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

2 **(i)** By sketching a suitable pair of graphs, show that the equation

$$\ln x = 2 - x^2$$

has only one root. [2]

(ii) Verify by calculation that this root lies between $x = 1.3$ and $x = 1.4$. [2]

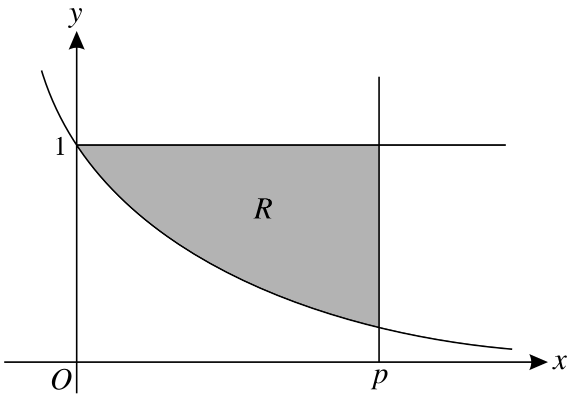
(iii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \sqrt{(2 - \ln x_n)}$$

converges, then it converges to the root of the equation in part **(i)**. [1]

(iv) Use the iterative formula $x_{n+1} = \sqrt{(2 - \ln x_n)}$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

3



The diagram shows the curve $y = e^{-x}$. The shaded region R is bounded by the curve and the lines $y = 1$ and $x = p$, where p is a constant.

(i) Find the area of R in terms of p. [4]

(ii) Show that if the area of R is equal to 1 then

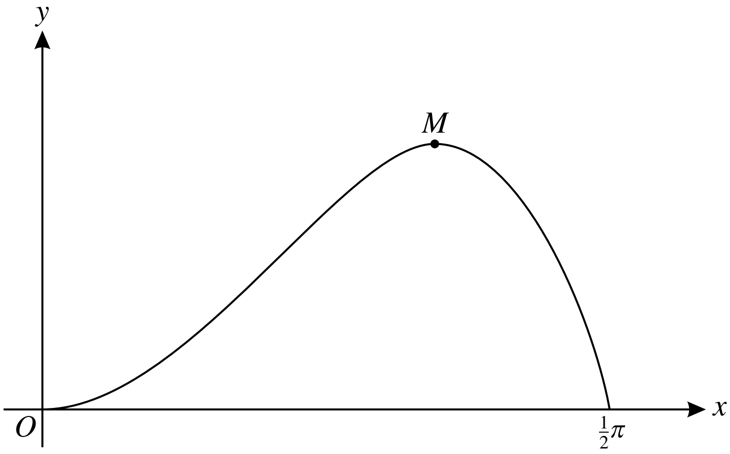
$$p = 2 - e^{-p}.$$ [1]

(iii) Use the iterative formula

$$p_{n+1} = 2 - e^{-p_n},$$

with initial value $p_1 = 2$, to calculate the value of p correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

4



The diagram shows the curve $y = x^2 \cos x$, for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.

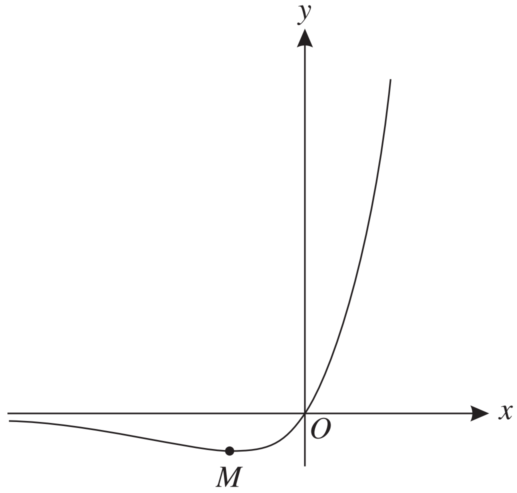
(i) Show by differentiation that the x-coordinate of M satisfies the equation

$$\tan x = \frac{2}{x}.$$ [4]

(ii) Verify by calculation that this equation has a root (in radians) between 1 and 1.2. [2]

(iii) Use the iterative formula $x_{n+1} = \tan^{-1}\left(\frac{2}{x_n}\right)$ to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

5



The diagram shows the curve $y = xe^{2x}$ and its minimum point M.

(i) Find the exact coordinates of M. [5]

(ii) Show that the curve intersects the line $y = 20$ at the point whose x-coordinate is the root of the equation

$$x = \frac{1}{2}\ln\left(\frac{20}{x}\right). \qquad [1]$$

(iii) Use the iterative formula

$$x_{n+1} = \frac{1}{2}\ln\left(\frac{20}{x_n}\right),$$

with initial value $x_1 = 1.3$, to calculate the root correct to 2 decimal places, giving the result of each iteration to 4 decimal places. [3]

6 **(i)** By sketching a suitable pair of graphs, show that the equation

$$\cos x = 2 - 2x,$$

where x is in radians, has only one root for $0 \leqslant x \leqslant \frac{1}{2}\pi$. [2]

(ii) Verify by calculation that this root lies between 0.5 and 1. [2]

(iii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = 1 - \frac{1}{2}\cos x_n$$

converges, then it converges to the root of the equation in part **(i)**. [1]

(iv) Use this iterative formula, with initial value $x_1 = 0.6$, to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]