Numerical Solutions of Equations

Question Paper 6

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Numerical Solutions of Equations
Sub Topic	
Booklet	Question Paper 6

Time Allowed: **44 minutes**

Score: **/36**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 **(i)** By sketching a suitable pair of graphs, show that there is only one value of x in the interval $0 < x < \frac{1}{2}\pi$ that is a root of the equation

$$\cot x = x.$$ [2]

(ii) Verify by calculation that this root lies between 0.8 and 0.9 radians. [2]

(iii) Show that this value of x is also a root of the equation

$$x = \tan^{-1}\left(\frac{1}{x}\right).$$ [1]

(iv) Use the iterative formula

$$x_{n+1} = \tan^{-1}\left(\frac{1}{x_n}\right)$$

to determine this root correct to 2 decimal places, showing the result of each iteration. [3]

2 The sequence of values given by the iterative formula

$$x_{n+1} = \frac{1}{5}\left(4x_n + \frac{306}{x_n^4}\right),$$

with initial value $x_1 = 3$, converges to α.

(i) Use this iterative formula to find α correct to 3 decimal places, showing the result of each iteration. [3]

(ii) State an equation satisfied by α, and hence show that the exact value of α is $\sqrt[5]{306}$. [2]

3 **(i)** By sketching a suitable pair of graphs, for $x < 0$, show that exactly one root of the equation $x^2 = 2^x$ is negative. [2]

(ii) Verify by calculation that this root lies between -1.0 and -0.5. [2]

(iii) Use the iterative formula

$$x_{n+1} = -\sqrt{\left(2^{x_n}\right)}$$

to determine this root correct to 2 significant figures, showing the result of each iteration. [3]

4 **(i)** By sketching a suitable pair of graphs, show that the equation

$$\ln x = 2 - x^2$$

has exactly one root. [3]

(ii) Verify by calculation that the root lies between 1.0 and 1.4. [2]

(iii) Use the iterative formula

$$x_{n+1} = \sqrt{(2 - \ln x_n)}$$

to determine the root correct to 2 decimal places, showing the result of each iteration. [3]

5 **(i)** By sketching a suitable pair of graphs, show that there is only one value of x in the interval $0 < x < \frac{1}{2}\pi$ that is a root of the equation

$$\sin x = \frac{1}{x^2}.$$ [2]

(ii) Verify by calculation that this root lies between 1 and 1.5. [2]

(iii) Show that this value of x is also a root of the equation

$$x = \sqrt{(\operatorname{cosec} x)}.$$ [1]

(iv) Use the iterative formula

$$x_{n+1} = \sqrt{(\operatorname{cosec} x_n)}$$

to determine this root correct to 3 significant figures, showing the value of each approximation that you calculate. [3]