

# Sequences & nth Term

## Question Paper 6

Level	IGCSE
Subject	Maths (0580)
Exam Board	Cambridge International Examinations (CIE)
Paper Type	Extended
Topic	Algebra and graphs
Sub-Topic	Sequences & nth Term
Booklet	Question Paper 6

**Time Allowed:** 59 minutes

**Score:** /49

**Percentage:** /100

**Grade Boundaries:**

A*	A	B	C	D	E	U
>85%	75%	60%	45%	35%	25%	<25%

1 The first four terms of a sequence are

$$T_1 = 1^2 \quad T_2 = 1^2 + 2^2 \quad T_3 = 1^2 + 2^2 + 3^2 \quad T_4 = 1^2 + 2^2 + 3^2 + 4^2.$$

(a) The  $n$ th term is given by  $T_n = \frac{1}{6} n(n+1)(2n+1)$ .

Work out the value of  $T_{23}$ .

*Answer(a)*  $T_{23} = \dots\dots\dots$  [2]

(b) A new sequence is formed as follows.

$$U_1 = T_2 - T_1 \quad U_2 = T_3 - T_2 \quad U_3 = T_4 - T_3 \quad \dots\dots$$

(i) Find the values of  $U_1$  and  $U_2$ .

*Answer(b)(i)*  $U_1 = \dots\dots\dots$  and  $U_2 = \dots\dots\dots$  [2]

(ii) Write down a formula for the  $n$ th term,  $U_n$ .

*Answer(b)(ii)*  $U_n = \dots\dots\dots$  [1]

(c) The first four terms of another sequence are

$$V_1 = 2^2 \quad V_2 = 2^2 + 4^2 \quad V_3 = 2^2 + 4^2 + 6^2 \quad V_4 = 2^2 + 4^2 + 6^2 + 8^2.$$

By comparing this sequence with the one in **part (a)**, find a formula for the  $n$ th term,  $V_n$ .

*Answer(c)*  $V_n = \dots\dots\dots$  [2]

2 A sequence is given by  $u_1 = \sqrt{1}$ ,  $u_2 = \sqrt{3}$ ,  $u_3 = \sqrt{5}$ ,  $u_4 = \sqrt{7}$ , ...

(a) Find a formula for  $u_n$ , the  $n$ th term.

*Answer(a)*  $u_n = \dots\dots\dots$  [2]

(b) Find  $u_{29}$ .

*Answer(b)*  $u_{29} = \dots\dots\dots$  [1]

3 (a) The  $n$ th term of a sequence is  $n(n+1)$ .

(i) Write the two missing terms in the spaces. 2, 6, ....., 20, ..... [2]

(ii) Write down an expression in terms of  $n$  for the  $(n + 1)$ th term.

Answer(a)(ii) ..... [1]

(iii) The difference between the  $n$ th term and the  $(n + 1)$ th term is  $pn + q$ .

Find the values of  $p$  and  $q$ .

Answer(a)(iii)  $p =$  .....

$q =$  ..... [2]

(iv) Find the positions of the two consecutive terms which have a difference of 140.

Answer(a)(iv) ..... and ..... [2]

(b) A sequence  $u_1, u_2, u_3, u_4, \dots$  is given by the following rules.

$$u_1 = 2, \quad u_2 = 3 \quad \text{and} \quad u_n = 2u_{n-2} + u_{n-1} \quad \text{for } n \geq 3.$$

For example, the third term is  $u_3$  and  $u_3 = 2u_1 + u_2 = 2 \times 2 + 3 = 7$ .

So, the sequence is 2, 3, 7,  $u_4$ ,  $u_5$ , .....

(i) Show that  $u_4 = 13$ .

Answer(b)(i) ..... [1]

(ii) Find the value of  $u_5$ .

Answer(b)(ii)  $u_5 =$  ..... [1]

(iii) Two consecutive terms of the sequence are 3413 and 6827.

Find the term before and the term after these two given terms.

Answer(b)(iii) ....., 3413, ..... [2]

4 The first and the  $n$ th terms of sequences  $A$ ,  $B$  and  $C$  are shown in the table below.

(a) Complete the table for each sequence.

	1st term	2nd term	3rd term	4th term	5th term	$n$ th term
Sequence $A$	1					$n^3$
Sequence $B$	4					$4n$
Sequence $C$	4					$(n + 1)^2$

[5]

(b) Find

(i) the 8th term of sequence  $A$ ,

Answer(b)(i) ..... [1]

(ii) the 12th term of sequence  $C$ .

Answer(b)(ii) ..... [1]

(c) (i) Which term in sequence  $A$  is equal to 15 625?

Answer(c)(i) ..... [1]

(ii) Which term in sequence  $C$  is equal to 10 000?

Answer(c)(ii) ..... [1]

(d) The first four terms of sequences  $D$  and  $E$  are shown in the table below.

Use the results from **part (a)** to find the 5th and the  $n$ th terms of the sequences  $D$  and  $E$ .

	1st term	2nd term	3rd term	4th term	5th term	$n$ th term
Sequence $D$	5	16	39	80		
Sequence $E$	0	1	4	9		

[4]

5 (a) (i) Work out the first 3 terms of the sequence whose  $n$ th term is  $n(n + 2)$ .

Answer(a)(i) ..... , ..... , ..... [2]

(ii) Which term in this sequence is equal to 168?

Answer(a)(ii) ..... [3]

(b) Find a formula for the  $n$ th term of the following sequences.

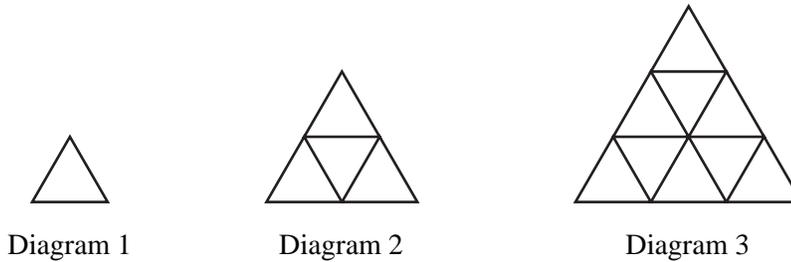
(i) 5      8      11      14      17 .....

Answer(b)(i) ..... [2]

(ii) 1      2      4      8      16 .....

Answer(b)(ii) ..... [2]

(c)



A sequence of diagrams is formed by drawing equilateral triangles each of side one centimetre.  
 Diagram 1 has 3 one centimetre lines.  
 Diagram 2 has 9 one centimetre lines.

The formula for the **total** number of one centimetre lines needed to draw **all of the first  $n$  diagrams** is

$$an^3 + bn^2 + n.$$

Find the values of  $a$  and  $b$ .

Answer(c)  $a =$  .....

$b =$  ..... [6]