

Differentiation – Log, Exponential & Trig Functions Question Paper 6

Level	A Level
Subject	Maths
Exam Board	OCR - MEI
Module	Core 3
Topic	Calculus
Sub Topic	Differentiation – Log, Exponential & Trig Functions
Booklet	Question Paper 6

Time Allowed: 63 minutes

Score: /52

Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	77.5%	70%	62.5%	57.5%	45%	<45%

1 The function $f(x)$ is defined by $f(x) = \sqrt{4 - x^2}$ for $-2 \leq x \leq 2$.

- (i) Show that the curve $y = \sqrt{4 - x^2}$ is a semicircle of radius 2, and explain why it is not the whole of this circle. [3]

Fig. 9 shows a point $P(a, b)$ on the semicircle. The tangent at P is shown.

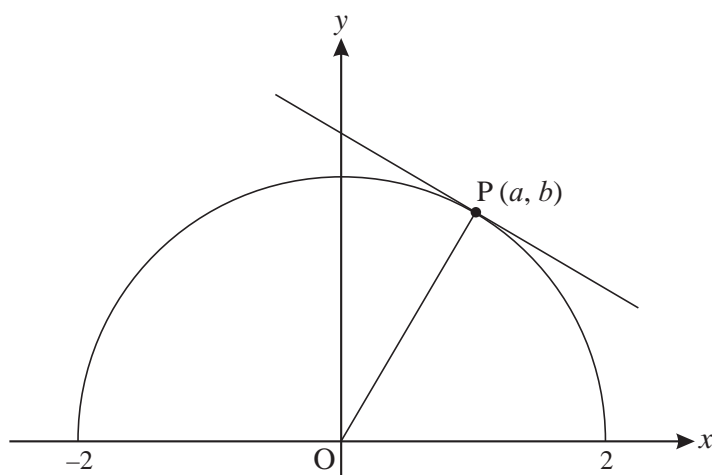


Fig. 9

- (ii) (A) Use the gradient of OP to find the gradient of the tangent at P in terms of a and b .

(B) Differentiate $\sqrt{4 - x^2}$ and deduce the value of $f'(a)$.

(C) Show that your answers to parts (A) and (B) are equivalent. [6]

The function $g(x)$ is defined by $g(x) = 3f(x - 2)$, for $0 \leq x \leq 4$.

- (iii) Describe a sequence of two transformations that would map the curve $y = f(x)$ onto the curve $y = g(x)$.

Hence sketch the curve $y = g(x)$. [6]

- (iv) Show that if $y = g(x)$ then $9x^2 + y^2 = 36x$. [3]

- 2 Fig. 7 shows part of the curve $y = f(x)$, where $f(x) = x\sqrt{1+x}$. The curve meets the x -axis at the origin and at the point P.

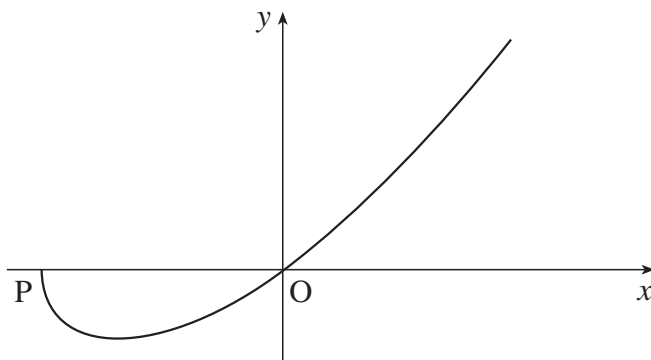


Fig. 7

- (i) Verify that the point P has coordinates $(-1, 0)$. Hence state the domain of the function $f(x)$. [2]
- (ii) Show that $\frac{dy}{dx} = \frac{2+3x}{2\sqrt{1+x}}$. [4]
- (iii) Find the exact coordinates of the turning point of the curve. Hence write down the range of the function. [4]
- (iv) Use the substitution $u = 1 + x$ to show that

$$\int_{-1}^0 x\sqrt{1+x} \, dx = \int_0^1 \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right) du.$$

Hence find the area of the region enclosed by the curve and the x -axis. [8]

- 3 Fig. 7 shows the curve $y = \frac{x^2}{1 + 2x^3}$. It is undefined at $x = a$; the line $x = a$ is a vertical asymptote.

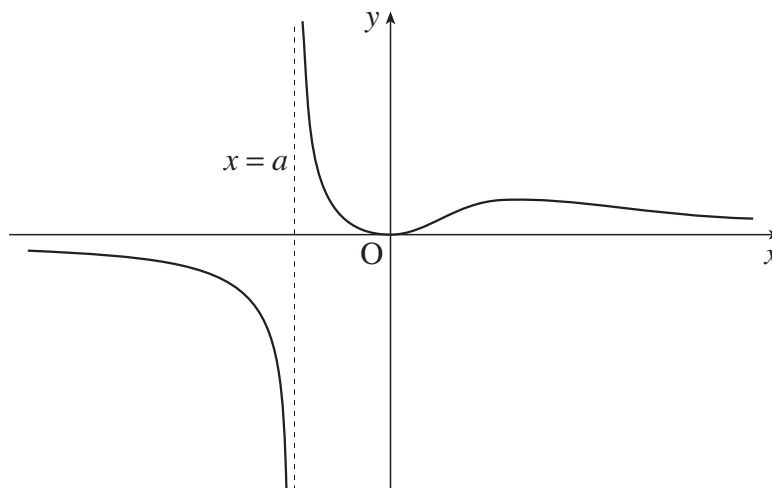


Fig. 7

- (i) Calculate the value of a , giving your answer correct to 3 significant figures. [3]
- (ii) Show that $\frac{dy}{dx} = \frac{2x - 2x^4}{(1 + 2x^3)^2}$. Hence determine the coordinates of the turning points of the curve. [8]
- (iii) Show that the area of the region between the curve and the x -axis from $x = 0$ to $x = 1$ is $\frac{1}{6} \ln 3$. [5]