Integration – Log, Exponential & Trig Functions

Question Paper 1

Level	A Level
Subject	Maths
Exam Board	OCR - MEI
Module	Core 3
Topic	Calculus
Sub Topic	Integration – Log, Exponential & Trig Functions
Booklet	Question Paper 1

Time Allowed: **58 minutes**

Score: **/48**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	77.5%	70%	62.5%	57.5%	45%	<45%

1 Find $\int \sqrt[3]{2x-1}\,\mathrm{d}x$. **[4]**

2 Fig. 8 shows the line $y = 1$ and the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{(x-2)^2}{x}$. The curve touches the x-axis at P(2, 0) and has another turning point at the point Q.

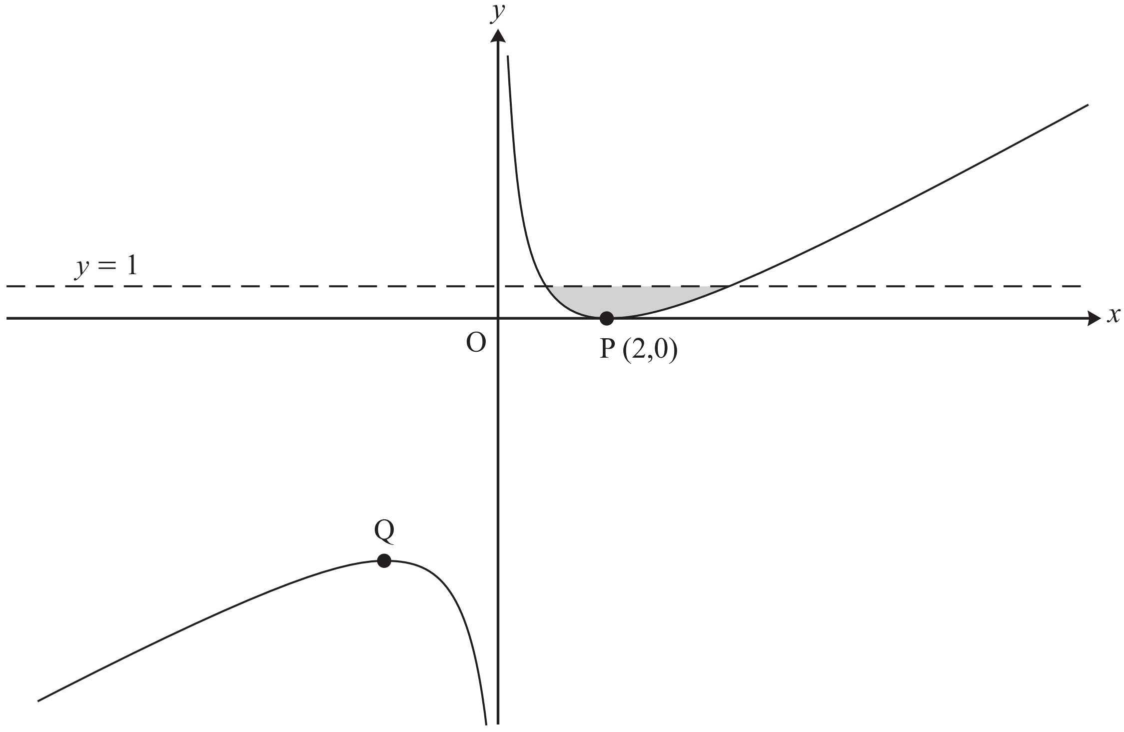

Fig. 8

(i) Show that $\mathrm{f}'(x) = 1 - \dfrac{4}{x^2}$, and find $\mathrm{f}''(x)$.

Hence find the coordinates of Q and, using $\mathrm{f}''(x)$, verify that it is a maximum point. **[7]**

(ii) Verify that the line $y = 1$ meets the curve $y = \mathrm{f}(x)$ at the points with x-coordinates 1 and 4. Hence find the exact area of the shaded region enclosed by the line and the curve. **[6]**

The curve $y = \mathrm{f}(x)$ is now transformed by a translation with vector $\begin{pmatrix} -1 \\ -1 \end{pmatrix}$. The resulting curve has equation $y = \mathrm{g}(x)$.

(iii) Show that $\mathrm{g}(x) = \dfrac{x^2 - 3x}{x+1}$. **[3]**

(iv) Without further calculation, write down the value of $\int_0^3 \mathrm{g}(x)\,\mathrm{d}x$, justifying your answer. **[2]**

3 Evaluate $\int_0^{\frac{1}{6}\pi}(1-\sin 3x)\,dx$, giving your answer in exact form. **[3]**

4 Fig. 9 shows the curve $y = xe^{-2x}$ together with the straight line $y = mx$, where m is a constant, with $0 < m < 1$. The curve and the line meet at O and P. The dashed line is the tangent at P.

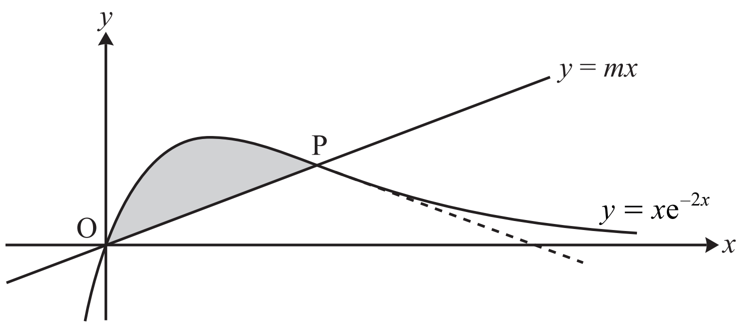

Fig. 9

(i) Show that the x-coordinate of P is $-\frac{1}{2}\ln m$. **[3]**

(ii) Find, in terms of m, the gradient of the tangent to the curve at P. **[4]**

You are given that OP and this tangent are equally inclined to the x-axis.

(iii) Show that $m = e^{-2}$, and find the exact coordinates of P. **[4]**

(iv) Find the exact area of the shaded region between the line OP and the curve. **[7]**

5 Using a suitable substitution or otherwise, show that $\int_0^{\frac{1}{2}\pi}\frac{\sin 2x}{3+\cos 2x}\,dx = \frac{1}{2}\ln 2$. **[5]**