

Integration – Log, Exponential & Trig Functions Question Paper 3

Level	A Level
Subject	Maths
Exam Board	OCR - MEI
Module	Core 3
Topic	Calculus
Sub Topic	Integration – Log, Exponential & Trig Functions
Booklet	Question Paper 3

Time Allowed: 53 minutes

Score: /44

Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	77.5%	70%	62.5%	57.5%	45%	<45%

1 Show that $\int_1^2 \frac{1}{\sqrt{3x-2}} dx = \frac{2}{3}$. [5]

- 2 Fig. 9 shows the curve $y = f(x)$, which has a y-intercept at $P(0, 3)$, a minimum point at $Q(1, 2)$, and an asymptote $x = -1$.

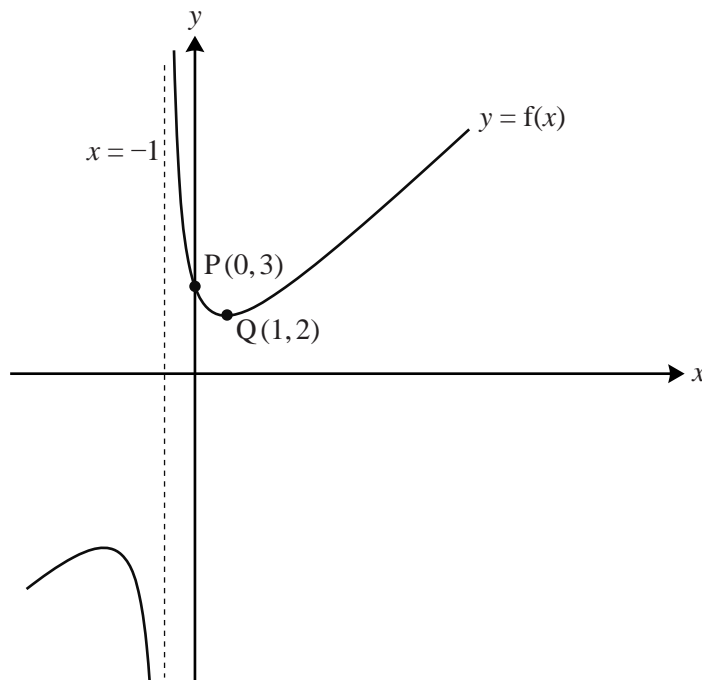


Fig. 9

- (i) Find the coordinates of the images of the points P and Q when the curve $y = f(x)$ is transformed to

(A) $y = 2f(x)$,

(B) $y = f(x + 1) + 2$.

[4]

You are now given that $f(x) = \frac{x^2 + 3}{x + 1}$, $x \neq -1$.

- (ii) Find $f'(x)$, and hence find the coordinates of the other turning point on the curve $y = f(x)$.

[6]

(iii) Show that $f(x - 1) = x - 2 + \frac{4}{x}$.

[3]

(iv) Find $\int_a^b \left(x - 2 + \frac{4}{x}\right) dx$ in terms of a and b .

Hence, by choosing suitable values for a and b , find the exact area enclosed by the curve $y = f(x)$, the x-axis, the y-axis and the line $x = 1$.

[5]

3 Evaluate $\int_0^{\frac{1}{6}\pi} \sin 3x \, dx$.

[3]

4 Fig. 8 shows the curve $y = f(x)$, where $f(x) = \frac{1}{1 + \cos x}$, for $0 \leq x \leq \frac{1}{2}\pi$.

P is the point on the curve with x -coordinate $\frac{1}{3}\pi$.

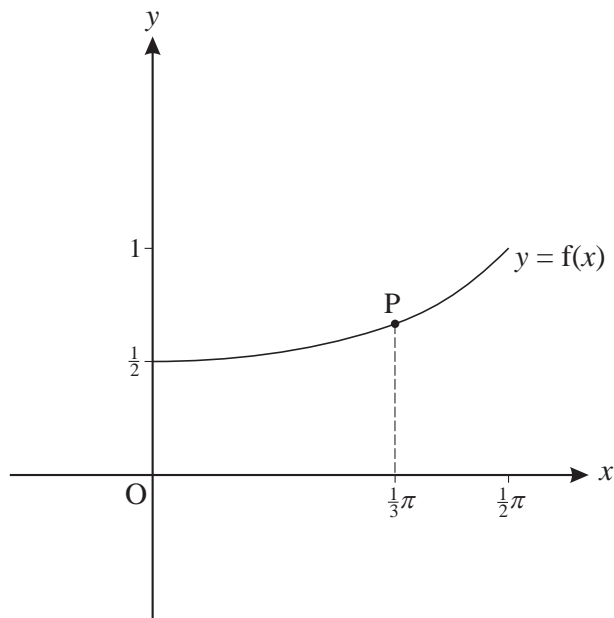


Fig. 8

- (i) Find the y -coordinate of P. [1]
- (ii) Find $f'(x)$. Hence find the gradient of the curve at the point P. [5]
- (iii) Show that the derivative of $\frac{\sin x}{1 + \cos x}$ is $\frac{1}{1 + \cos x}$. Hence find the exact area of the region enclosed by the curve $y = f(x)$, the x -axis, the y -axis and the line $x = \frac{1}{3}\pi$. [7]
- (iv) Show that $f^{-1}(x) = \arccos\left(\frac{1}{x} - 1\right)$. State the domain of this inverse function, and add a sketch of $y = f^{-1}(x)$ to a copy of Fig. 8. [5]