

# Integration by Parts

## Question Paper 4

<b>Level</b>	A Level
<b>Subject</b>	Maths
<b>Exam Board</b>	OCR - MEI
<b>Module</b>	Core 3
<b>Topic</b>	Calculus
<b>Sub Topic</b>	Integration by Parts
<b>Booklet</b>	Question Paper 4

**Time Allowed:** 60 minutes

**Score:** /50

**Percentage:** /100

**Grade Boundaries:**

A*	A	B	C	D	E	U
>85%	77.5%	70%	62.5%	57.5%	45%	<45%

- 1 Find the exact value of  $\int_0^2 \sqrt{1+4x} \, dx$ , showing your working. [5]

- 2 Fig. 8 shows the line  $y = x$  and parts of the curves  $y = f(x)$  and  $y = g(x)$ , where

$$f(x) = e^{x-1}, \quad g(x) = 1 + \ln x.$$

The curves intersect the axes at the points A and B, as shown. The curves and the line  $y = x$  meet at the point C.

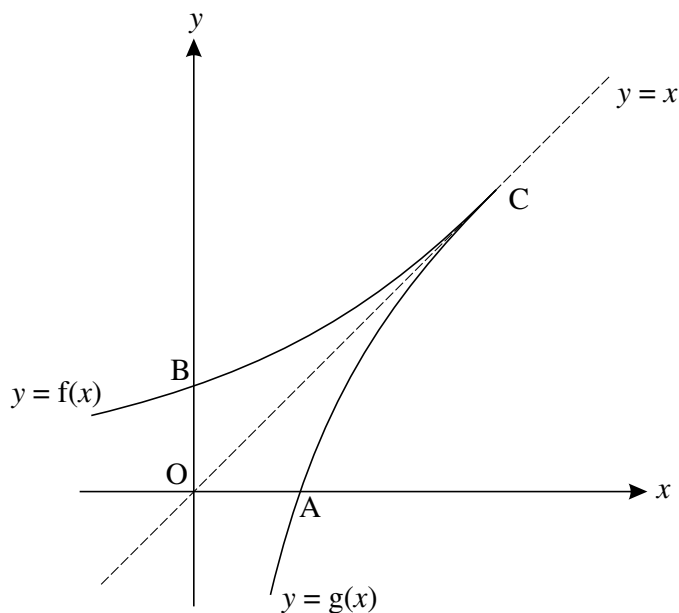


Fig. 8

- (i) Find the exact coordinates of A and B. Verify that the coordinates of C are (1, 1). [5]
- (ii) Prove algebraically that  $g(x)$  is the inverse of  $f(x)$ . [2]
- (iii) Evaluate  $\int_0^1 f(x) \, dx$ , giving your answer in terms of  $e$ . [3]
- (iv) Use integration by parts to find  $\int \ln x \, dx$ .  
Hence show that  $\int_{e^{-1}}^1 g(x) \, dx = \frac{1}{e}$ . [6]
- (v) Find the area of the region enclosed by the lines OA and OB, and the arcs AC and BC. [2]

3 A curve is defined by the equation  $y = 2x \ln(1 + x)$ .

(i) Find  $\frac{dy}{dx}$  and hence verify that the origin is a stationary point of the curve. [4]

(ii) Find  $\frac{d^2y}{dx^2}$ , and use this to verify that the origin is a minimum point. [5]

(iii) Using the substitution  $u = 1 + x$ , show that  $\int \frac{x^2}{1+x} dx = \int \left(u - 2 + \frac{1}{u}\right) du$ .

Hence evaluate  $\int_0^1 \frac{x^2}{1+x} dx$ , giving your answer in an exact form. [6]

(iv) Using integration by parts and your answer to part (iii), evaluate  $\int_0^1 2x \ln(1+x) dx$ . [4]

4 Find  $\int xe^{3x} dx$ . [4]

5 Show that  $\int_1^4 \frac{x}{x^2+2} dx = \frac{1}{2} \ln 6$ . [4]