

# Motion of a Projectile

## Question Paper 6

|            |                        |
|------------|------------------------|
| Level      | International A Level  |
| Subject    | Maths                  |
| Exam Board | CIE                    |
| Topic      | Motion of a Projectile |
| Sub Topic  |                        |
| Booklet    | Question Paper 6       |

Time Allowed: 57 minutes

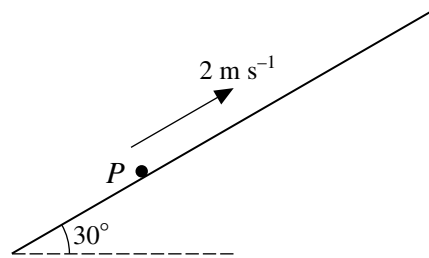
Score: /47

Percentage: /100

Grade Boundaries:

| A*   | A      | B   | C     | D     | E   | U    |
|------|--------|-----|-------|-------|-----|------|
| >85% | '77.5% | 70% | 62.5% | 57.5% | 45% | <45% |

1



A particle  $P$  of mass  $0.2 \text{ kg}$  is projected with velocity  $2 \text{ m s}^{-1}$  upwards along a line of greatest slope on a plane inclined at  $30^\circ$  to the horizontal (see diagram). Air resistance of magnitude  $0.5v \text{ N}$  opposes the motion of  $P$ , where  $v \text{ m s}^{-1}$  is the velocity of  $P$  at time  $t \text{ s}$  after projection. The coefficient of friction between  $P$  and the plane is  $\frac{1}{2\sqrt{3}}$ . The particle  $P$  reaches a position of instantaneous rest when  $t = T$ .

(i) Show that, while  $P$  is moving up the plane,  $\frac{dv}{dt} = -2.5(3 + v)$ . [3]

(ii) Calculate  $T$ . [4]

(iii) Calculate the speed of  $P$  when  $t = 2T$ . [5]

2 A particle is projected from a point  $O$  on horizontal ground. The velocity of projection has magnitude  $20 \text{ m s}^{-1}$  and direction upwards at an angle  $\theta$  to the horizontal. The particle passes through the point which is  $7 \text{ m}$  above the ground and  $16 \text{ m}$  horizontally from  $O$ , and hits the ground at the point  $A$ .

(i) Using the equation of the particle's trajectory and the identity  $\sec^2 \theta = 1 + \tan^2 \theta$ , show that the possible values of  $\tan \theta$  are  $\frac{3}{4}$  and  $\frac{17}{4}$ . [4]

(ii) Find the distance  $OA$  for each of the two possible values of  $\tan \theta$ . [3]

(iii) Sketch in the same diagram the two possible trajectories. [2]

- 3 A particle is projected horizontally with speed  $12 \text{ m s}^{-1}$  from the top of a high cliff. Find the direction of motion of the particle after 2 s. [3]
- 4 Two particles  $P$  and  $Q$  are projected simultaneously with speed  $40 \text{ m s}^{-1}$  from a point  $O$  on a horizontal plane. Both particles subsequently pass at different times through the point  $A$  which has horizontal and vertically upward displacements from  $O$  of 40 m and 15 m respectively.
- satisfies the equation  $\tan^2 \theta - 8 \tan \theta + 4 = 0$ . [3]
- (ii) Calculate the distance between the points at which  $P$  and  $Q$  strike the plane. [5]
- 5 A particle is projected from a point  $O$  with speed  $V \text{ m s}^{-1}$  at an angle  $\theta$  above the horizontal. After 0.3 s the particle is moving with speed  $25 \text{ m s}^{-1}$  at an angle  $\tan^{-1}\left(\frac{7}{24}\right)$  above the horizontal.
- (i) Show that  $V \cos \theta = 24$ . [2]
- (ii) Find the value of  $V \sin \theta$ , and hence find  $V$  and  $\theta$ . [5]
- 6 A particle  $P$  of mass 0.1 kg is projected vertically upwards from a point  $O$  with speed  $20 \text{ m s}^{-1}$ . Air resistance of magnitude  $0.1v \text{ N}$  opposes the motion, where  $v \text{ m s}^{-1}$  is the speed of  $P$  at time  $t$  s after projection.
- (i) Show that, while  $P$  is moving upwards,  $\frac{1}{v+10} \frac{dv}{dt} = -1$ . [2]
- (ii) Hence find an expression for  $v$  in terms of  $t$ , and explain why it is valid only for  $0 \leq t \leq \ln 3$ . [6]
- (iii) Find the initial acceleration of  $P$ . [2]