Equilibrium of a Rigid Body

Question Paper 1

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Equilibrium of a Rigid Body
Sub Topic	
Booklet	Question Paper 1

Time Allowed: **58 minutes**

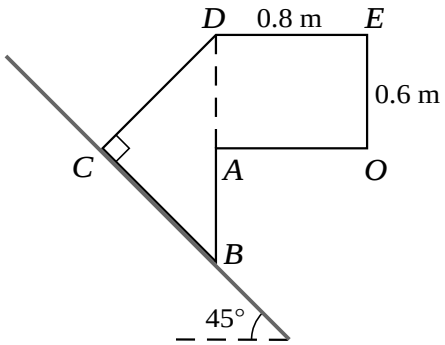
Score: **/48**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1



The diagram shows the cross-section $OABCDE$ through the centre of mass of a uniform prism on a rough inclined plane. The portion $ADEO$ is a rectangle in which $AD = OE = 0.6$ m and $DE = AO = 0.8$ m; the portion BCD is an isosceles triangle in which angle BCD is a right angle, and A is the mid-point of BD. The plane is inclined at 45° to the horizontal, BC lies along a line of greatest slope of the plane and DE is horizontal.

(i) Calculate the distance of the centre of mass of the prism from BD. [3]

The weight of the prism is 21 N, and it is held in equilibrium by a horizontal force of magnitude P N acting along ED.

(ii) **(a)** Find the smallest value of P for which the prism does not topple. [2]

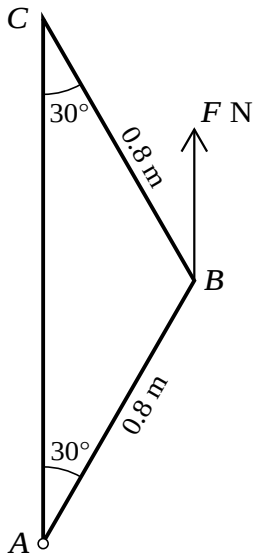
(b) It is given that the prism is about to slip for this smallest value of P. Calculate the coefficient of friction between the prism and the plane. [3]

The value of P is gradually increased until the prism ceases to be in equilibrium.

(iii) Show that the prism topples before it begins to slide, stating the value of P at which equilibrium is broken. [5]

2 A triangular frame ABC consists of two uniform rigid rods each of length 0.8 m and weight 3 N, and a longer uniform rod of weight 4 N. The triangular frame has $AB = BC$, and angle BAC = angle $BCA = 30°$.

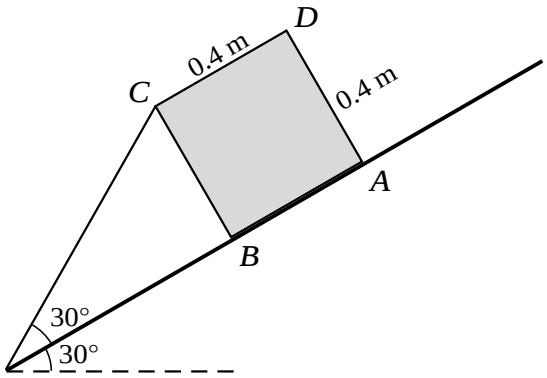
(i) Calculate the distance of the centre of mass of the frame from AC. [3]



The vertex A of the frame is attached to a smooth hinge at a fixed point. The frame is held in equilibrium with AC vertical by a vertical force of magnitude F N applied to the frame at B (see diagram).

(ii) Calculate F, and state the magnitude and direction of the force acting on the frame at the hinge. [3]

3



A uniform solid cube with edges of length 0.4 m rests in equilibrium on a rough plane inclined at an angle of 30° to the horizontal. $ABCD$ is a cross-section through the centre of mass of the cube, with AB along a line of greatest slope. B lies below the level of A. One end of a light elastic string with modulus of elasticity 12 N and natural length 0.4 m is attached to C. The other end of the string is attached to a point below the level of B on the same line of greatest slope, such that the string makes an angle of 30° with the plane (see diagram). The cube is on the point of toppling. Find

(i) the tension in the string, [3]

(ii) the weight of the cube. [4]

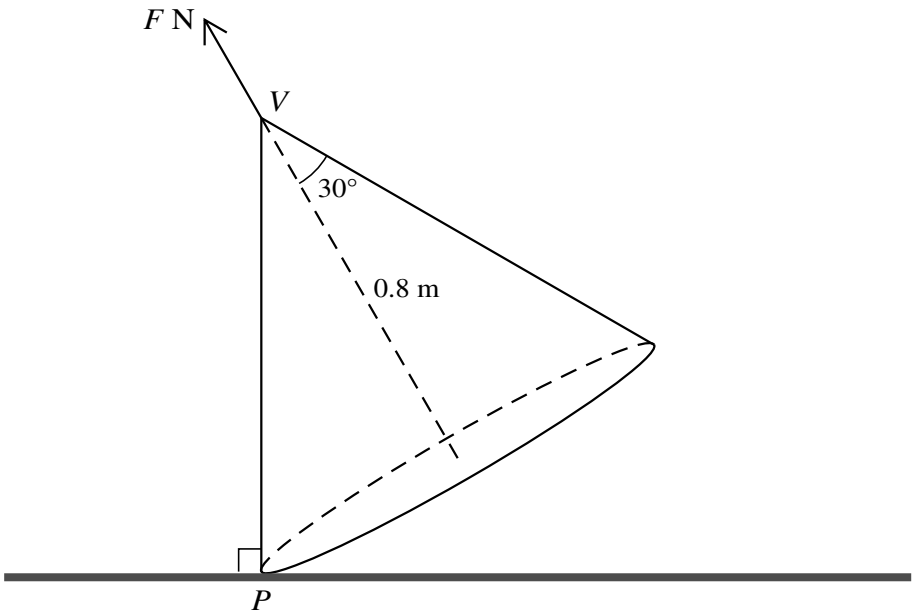
4 A uniform semicircular lamina has diameter AB of length 0.8 m.

(i) Find the distance of the centre of mass of the lamina from AB. [2]

The lamina rests in a vertical plane, with the point B of the lamina in contact with a rough horizontal surface and with A vertically above B. Equilibrium is maintained by a force of magnitude 6 N in the plane of the lamina, applied to the lamina at A and acting at an angle of 20° below the horizontal.

(ii) Calculate the mass of the lamina. [3]

5

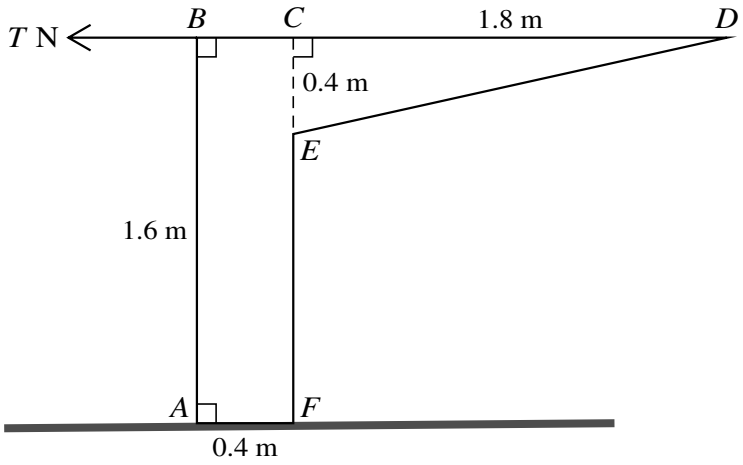


A uniform solid cone with height 0.8 m and semi-vertical angle 30° has weight 20 N. The cone rests in equilibrium with a single point P of its base in contact with a rough horizontal surface, and its vertex V vertically above P. Equilibrium is maintained by a force of magnitude F N acting along the axis of symmetry of the cone and applied to V (see diagram).

(i) Show that the moment of the weight of the cone about P is 6 N m. [2]

(ii) Hence find F. [2]

6



ABCDEF is the cross-section through the centre of mass of a uniform solid prism. *ABCF* is a rectangle in which $AB = CF = 1.6$ m, and $BC = AF = 0.4$ m. *CDE* is a triangle in which $CD = 1.8$ m, $CE = 0.4$ m, and angle $DCE = 90°$. The prism stands on a rough horizontal surface. A horizontal force of magnitude T N acts at B in the direction CB (see diagram). The prism is in equilibrium.

(i) Show that the distance of the centre of mass of the prism from *AB* is 0.488 m. [4]

(ii) Given that the weight of the prism is 100 N, find the greatest and least possible values of T. [3]

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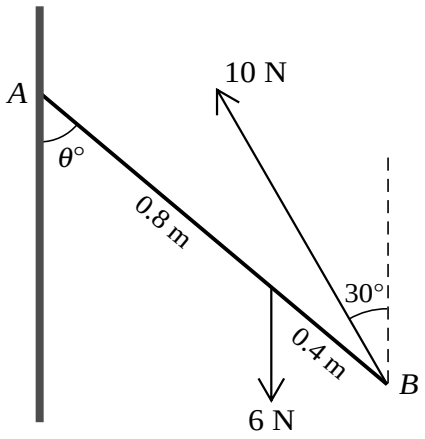

A non-uniform rod *AB* of weight 6 N rests in limiting equilibrium with the end *A* in contact with a rough vertical wall. $AB = 1.2$ m, the centre of mass of the rod is 0.8 m from *A*, and the angle between *AB* and the downward vertical is $\theta°$. A force of magnitude 10 N acting at an angle of 30° to the upwards vertical is applied to the rod at *B* (see diagram). The rod and the line of action of the 10 N force lie in a vertical plane perpendicular to the wall. Calculate

(i) the value of θ, [4]

(ii) the coefficient of friction between the rod and the wall. [2]