Equilibrium of a Rigid Body

Question Paper 10

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Equilibrium of a Rigid Body
Sub Topic	
Booklet	Question Paper 10

Time Allowed: **87 minutes**

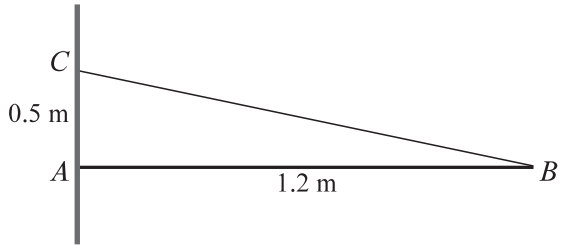
Score: **/72**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1

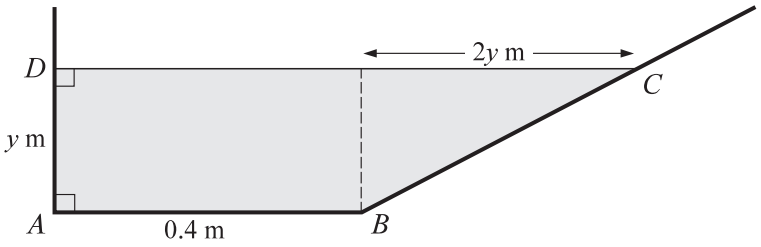


A uniform rod AB of length 1.2 m and weight 30 N is in equilibrium with the end A in contact with a vertical wall. AB is held at right angles to the wall by a light inextensible string. The string has one end attached to the rod at B and the other end attached to a point C of the wall. The point C is 0.5 m vertically above A (see diagram). Find

(i) the tension in the string, [3]

(ii) the horizontal and vertical components of the force exerted on the rod by the wall at A. [3]

2

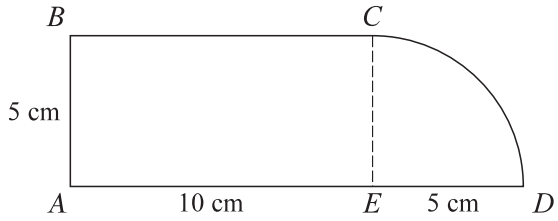


A light container has a vertical cross-section in the form of a trapezium. The container rests on a horizontal surface. Grain is poured into the container to a depth of y m. As shown in the diagram, the cross-section $ABCD$ of the grain is such that $AB = 0.4$ m and $DC = (0.4 + 2y)$ m.

(i) When $y = 0.3$, find the vertical height of the centre of mass of the grain above the base of the container. [5]

(ii) Find the value of y for which the container is about to topple. [5]

3



A uniform lamina $ABCDE$ consists of a rectangular part with sides 5 cm and 10 cm, and a part in the form of a quarter of a circle of radius 5 cm, as shown in the diagram.

(i) Show that the distance of the centre of mass of the part CDE of the lamina is $\frac{20}{3\pi}$ cm from CE. [2]

(ii) Find the distance of the centre of mass of the lamina $ABCDE$ from the edge AB. [4]

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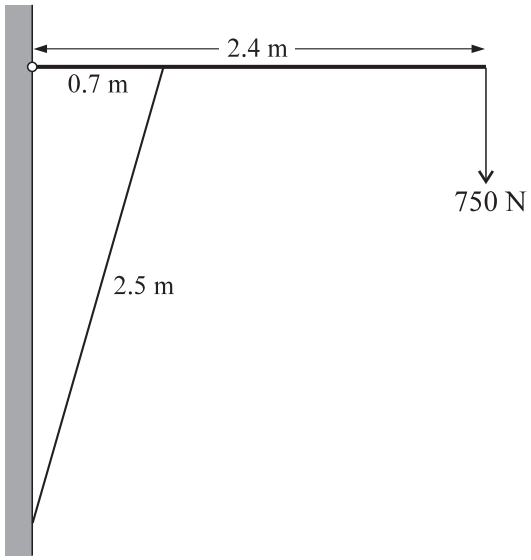

A uniform beam has length 2.4 m and weight 68 N. The beam is hinged at a fixed point of a vertical wall, and held in a horizontal position by a light rod of length 2.5 m. One end of the rod is attached to the beam at a point 0.7 m from the wall, and the other end of the rod is attached to the wall at a point vertically below the hinge. The beam carries a load of 750 N at its end (see diagram).

(i) Find the force in the rod. [4]

The components of the force exerted by the hinge on the beam are X N horizontally towards the wall and Y N vertically downwards.

(ii) Find the values of X and Y. [3]

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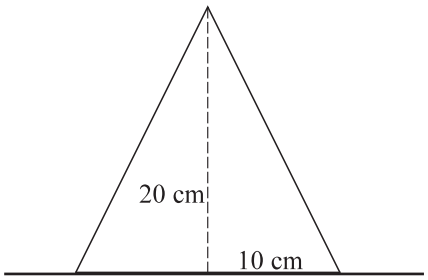

Fig. 1

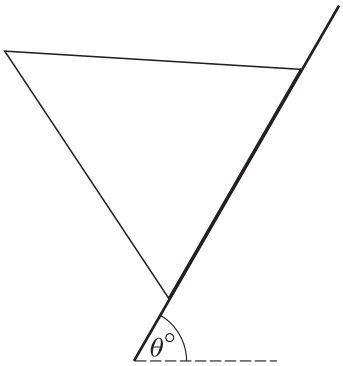

Fig. 2

A uniform solid cone has height 20 cm and base radius 10 cm. It is placed with its axis vertical on a rough horizontal plane (see Fig. 1). The plane is slowly tilted and the cone remains in equilibrium until the angle of inclination of the plane reaches $\theta°$, when the cone begins to topple without sliding (see Fig. 2).

(i) Find the value of θ. [3]

(ii) What can you say about the value of the coefficient of friction between the cone and the plane? [3]

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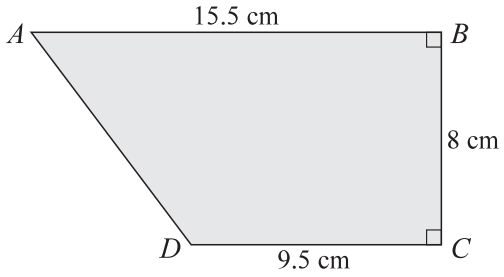

Fig. 1

Fig. 1 shows a uniform lamina $ABCD$ with dimensions $AB = 15.5$ cm, $BC = 8$ cm and $CD = 9.5$ cm. Angles ABC and BCD are right angles.

(i) Show that the distance of the centre of mass of the lamina from the side BC is 6.37 cm. [4]

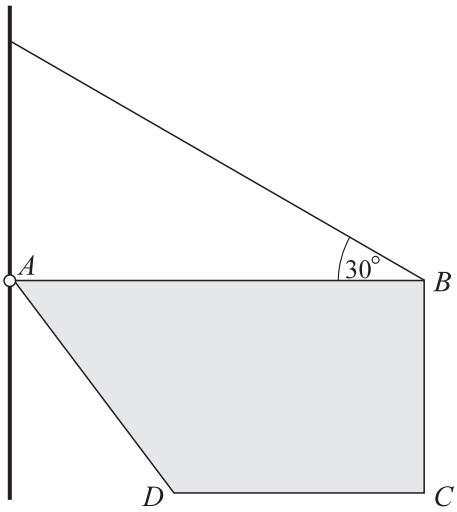

Fig. 2

The lamina is smoothly hinged to a wall at A and is supported, with AB horizontal, by a light wire attached at B. The other end of the wire is attached to a point on the wall, vertically above A, such that the wire makes an angle of 30° with AB (see Fig. 2). The mass of the lamina is 8 kg. Find

(ii) the tension in the wire, [4]

(iii) the magnitude of the vertical component of the force acting on the lamina at A. [2]

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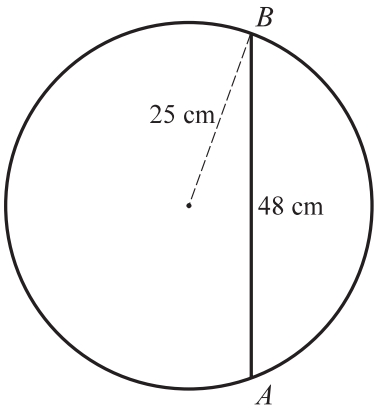

A frame consists of a uniform circular ring of radius 25 cm and mass 1.5 kg, and a uniform rod of length 48 cm and mass 0.6 kg. The ends A and B of the rod are attached to points on the circumference of the ring, as shown in the diagram. Find the distance of the centre of mass of the frame from the centre of the ring. [4]

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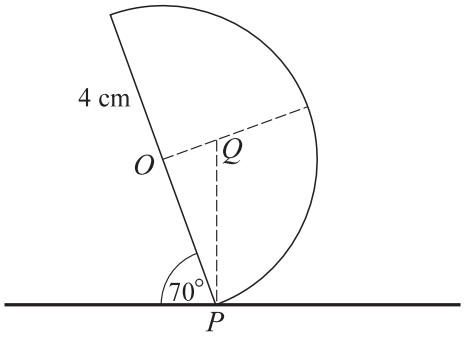

A uniform solid hemisphere, with centre O and radius 4 cm, is held so that a point P of its rim is in contact with a horizontal surface. The plane face of the hemisphere makes an angle of 70° with the horizontal. Q is the point on the axis of symmetry of the hemisphere which is vertically above P. The diagram shows the vertical cross-section of the hemisphere which contains O, P and Q. [3]

(i) Determine whether or not the centre of mass of the hemisphere is between O and Q.

The hemisphere is now released.

(ii) State whether or not the hemisphere falls on to its plane face, giving a reason for your answer. [2]

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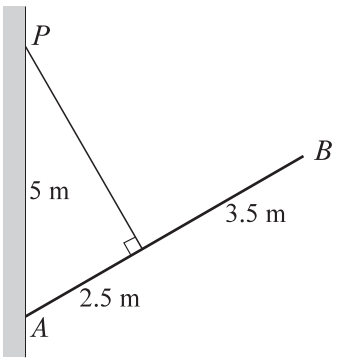

A uniform beam AB has length 6 m and mass 45 kg. One end of a light inextensible rope is attached to the beam at the point 2.5 m from A. The other end of the rope is attached to a fixed point P on a vertical wall. The beam is in equilibrium with A in contact with the wall at a point 5 m below P. The rope is taut and at right angles to AB (see diagram). Find

(i) the tension in the rope, [4]

(ii) the horizontal and vertical components of the force exerted by the wall on the beam at A. [3]

10 **(i)**

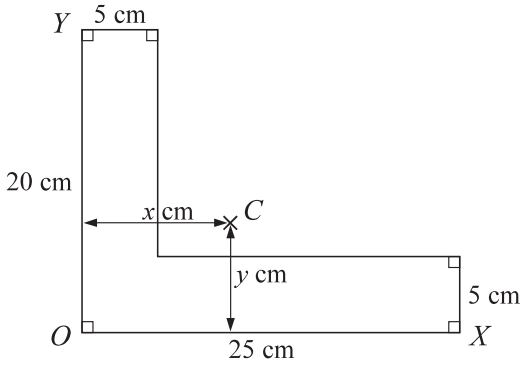

Fig. 1

Fig. 1 shows the cross section through the centre of mass C of a uniform L-shaped prism. C is x cm from OY and y cm from OX. Find the values of x and y. [4]

(ii)

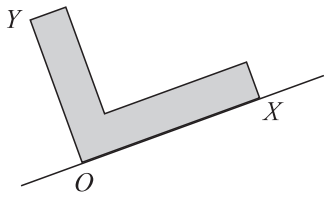

Fig. 2

The prism is placed on a rough plane with OX in contact with the plane. The plane is tilted from the horizontal so that OX lies along a line of greatest slope, as shown in Fig. 2. When the angle of inclination of the plane is sufficiently great the prism starts to slide (without toppling). Show that the coefficient of friction between the prism and the plane is less than $\frac{7}{5}$. [4]

(iii)

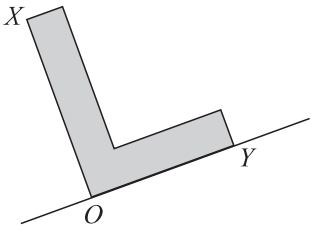

Fig. 3

The prism is now placed on a rough plane with OY in contact with the plane. The plane is tilted from the horizontal so that OY lies along a line of greatest slope, as shown in Fig. 3. When the angle of inclination of the plane is sufficiently great the prism starts to topple (without sliding). Find the least possible value of the coefficient of friction between the prism and the plane. [3]