Equilibrium of a Rigid Body

Question Paper 3

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Equilibrium of a Rigid Body
Sub Topic	
Booklet	Question Paper 3

Time Allowed: **56 minutes**

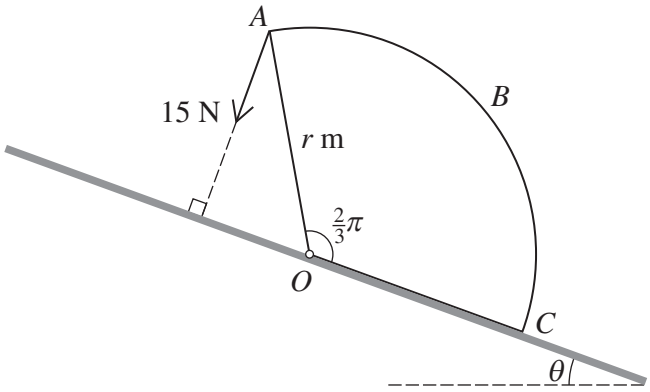
Score: **/46**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1



$OABC$ is the cross-section through the centre of mass of a uniform prism of weight 20 N. The cross-section is in the shape of a sector of a circle with centre O, radius $OA = r$ m and angle $AOC = \frac{2}{3}\pi$ radians. The prism lies on a plane inclined at an angle θ radians to the horizontal, where $\theta < \frac{1}{3}\pi$. OC lies along a line of greatest slope with O higher than C. The prism is freely hinged to the plane at O. A force of magnitude 15 N acts at A, in a direction towards to the plane and at right angles to it (see diagram). Given that the prism remains in equilibrium, find the set of possible values of θ. [9]

2 A uniform semicircular lamina of radius 0.25 m has diameter AB. It is freely suspended at A from a fixed point and hangs in equilibrium.

(i) Find the distance of the centre of mass of the lamina from the diameter AB. [1]

(ii) Calculate the angle which the diameter AB makes with the vertical. [2]

The lamina is now held in equilibrium with the diameter AB vertical by means of a force applied at B. This force has magnitude 6 N and acts at 45° to the upward vertical in the plane of the lamina.

(iii) Calculate the weight of the lamina. [3]

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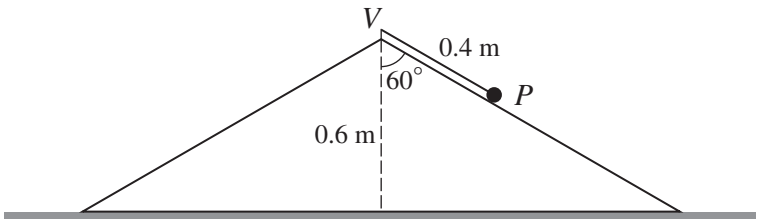

A uniform solid cone of height 0.6 m and mass 0.5 kg has its axis of symmetry vertical and its vertex V uppermost. The semi-vertical angle of the cone is 60° and the surface is smooth. The cone is fixed to a horizontal surface. A particle P of mass 0.2 kg is connected to V by a light inextensible string of length 0.4 m (see diagram).

(i) Calculate the height, above the horizontal surface, of the centre of mass of the cone with the particle. [3]

P is set in motion, and moves with angular speed 4 rad s^{-1} in a circular path on the surface of the cone.

(ii) Show that the tension in the string is 1.96 N, and calculate the magnitude of the force exerted on P by the cone. [5]

(iii) Find the speed of P. [1]

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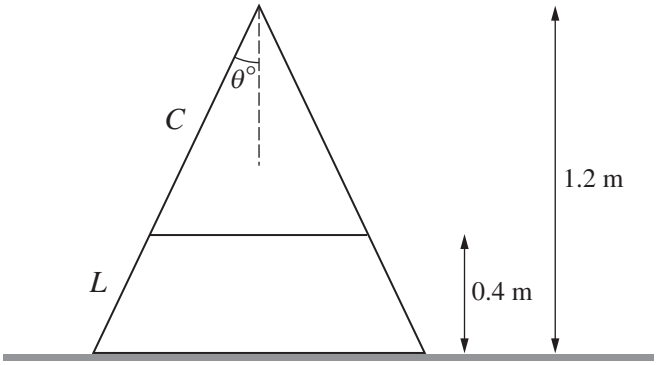

A uniform solid cone of height 1.2 m and semi-vertical angle $\theta°$ is divided into two parts by a cut parallel to and 0.4 m from the circular base. The upper conical part, C, has weight 16 N, and the lower part, L, has weight 38 N. The two parts of the solid rest in equilibrium with the larger plane face of L on a horizontal surface and the smaller plane face of L covered by the base of C (see diagram).

(i) Calculate the distance of the centre of mass of L from its larger plane face. [3]

An increasing horizontal force is applied to the vertex of C. Equilibrium is broken when the magnitude of this force first exceeds 4 N, and C begins to slide on L.

(ii) By considering the forces on C,

(a) find the coefficient of friction between C and L, [1]

(b) show that $\theta > 14.0$, correct to 3 significant figures. [2]

C is removed and L is placed with its curved surface on the horizontal surface.

(iii) Given that L is on the point of toppling, calculate θ. [3]

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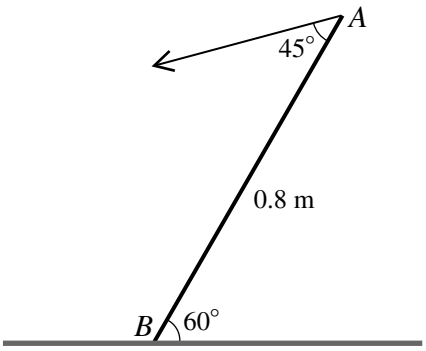

A uniform rod AB has weight 6 N and length 0.8 m. The rod rests in limiting equilibrium with B in contact with a rough horizontal surface and AB inclined at 60° to the horizontal. Equilibrium is maintained by a force, in the vertical plane containing AB, acting at A at an angle of 45° to AB (see diagram). Calculate

(i) the magnitude of the force applied at A, [3]

(ii) the least possible value of the coefficient of friction at B. [4]

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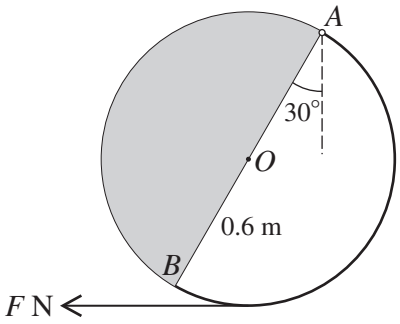

A circular object is formed from a uniform semicircular lamina of weight 12 N and a uniform semicircular arc of weight 8 N. The lamina and the arc both have centre O and radius 0.6 m and are joined at the ends of their common diameter AB. The object is freely pivoted to a fixed point at A with AB inclined at 30° to the vertical. The object is in equilibrium acted on by a horizontal force of magnitude F N applied at the lowest point of the object, and acting in the plane of the object (see diagram).

(i) Show that the centre of mass of the object is at O. [3]

(ii) Calculate F. [3]