Equilibrium of a Rigid Body

Question Paper 4

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Equilibrium of a Rigid Body
Sub Topic	
Booklet	Question Paper 4

Time Allowed: **62 minutes**

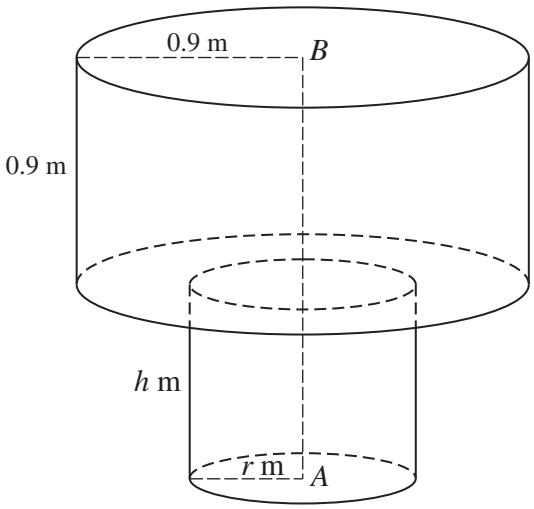
Score: **/51**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1



A cylinder of height 0.9 m and radius 0.9 m is placed symmetrically on top of a cylinder of height h m and radius r m, where $r < 0.9$, with plane faces in contact and axes in the same vertical line AB, where A and B are centres of plane faces of the cylinders (see diagram). Both cylinders are uniform and made of the same material. The lower cylinder is gradually tilted and when the axis of symmetry is inclined at 45° to the horizontal the upper cylinder is on the point of toppling without sliding.

(i) Find r. [2]

The upper cylinder is now fixed to the lower cylinder to create a uniform object.

(ii) Show that the centre of mass of the object is

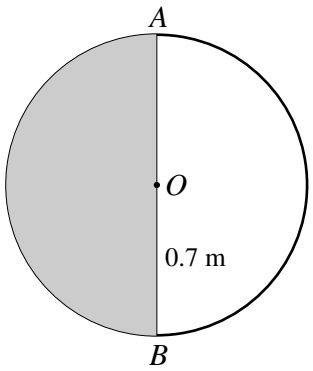
$$\frac{25h^2 + 180h + 81}{50h + 180} \text{ m}$$

from A. [3]

The object is placed with the plane face containing A in contact with a rough plane inclined at $\alpha°$ to the horizontal, where $\tan\alpha = 0.5$. The object is on the point of toppling without sliding.

(iii) Calculate h. [3]

2



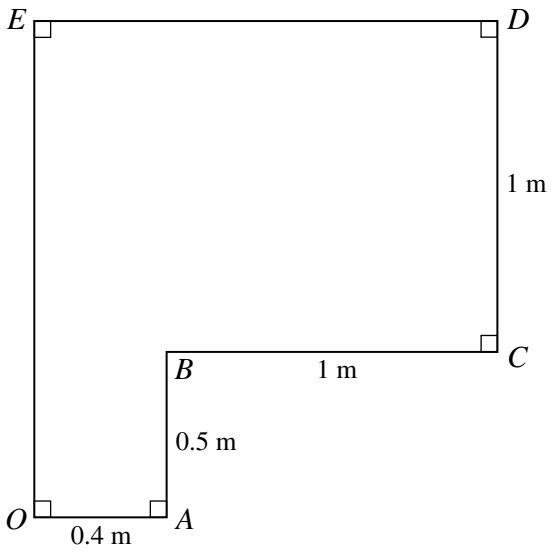
The diagram shows a circular object formed from a uniform semicircular lamina of weight 11 N and a uniform semicircular arc of weight 9 N. The lamina and the arc both have centre O and radius 0.7 m and are joined at the ends of their common diameter AB.

(i) Show that the distance of the centre of mass of the object from O is 0.0371 m, correct to 3 significant figures. [3]

The object hangs in equilibrium, freely suspended at A.

(ii) Find the angle between AB and the vertical and state whether the lowest point of the object is on the lamina or on the arc. [3]

3



The diagram shows the cross-section $OABCDE$ through the centre of mass of a uniform prism. The interior angles of the cross-section at O, A, C, D and E are all right angles. $OA = 0.4$ m, $AB = 0.5$ m and $BC = CD = 1$ m.

(i) Calculate the distance of the centre of mass of the prism from OE. [3]

The weight of the prism is 120 N. A force of magnitude F N acting along DE holds the prism in equilibrium when OA rests on a rough horizontal surface.

(ii) Find the set of possible values of F. [6]

4 A uniform hemispherical shell of weight 8 N and a uniform solid hemisphere of weight 12 N are joined along their circumferences to form a non-uniform sphere of radius 0.2 m.

(i) Show that the distance between the centre of mass of the sphere and the centre of the sphere is 0.005 m. [3]

This sphere is placed on a horizontal surface with its axis of symmetry horizontal. The equilibrium of the sphere is maintained by a force of magnitude F N acting parallel to the axis of symmetry applied to the highest point of the sphere.

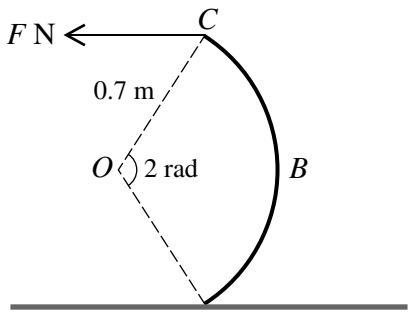
(ii) Calculate F. [3]

5 A ball is projected with velocity 25 m s^{-1} at an angle of 70° above the horizontal from a point O on horizontal ground. The ball subsequently bounces once on the ground at a point P before landing at a point Q where it remains at rest. The distance PQ is 17.1 m.

(i) Calculate the time taken by the ball to travel from O to P and the distance OP. [3]

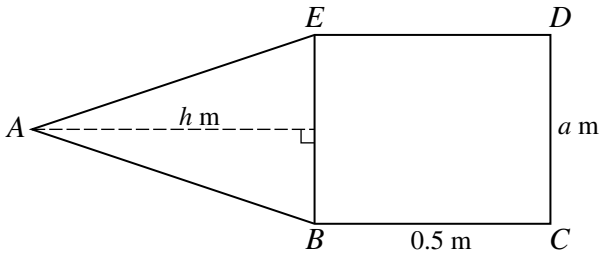
(ii) Given that the horizontal component of the velocity of the ball does not change at P, calculate the speed of the ball when it leaves P. [4]

6



The diagram shows a uniform object ABC of weight 3 N in the form of an arc of a circle with centre O and radius 0.7 m. The angle AOC is 2 radians. The object rests in equilibrium with A on a horizontal surface and C vertically above A. Equilibrium is maintained by a horizontal force of magnitude F N applied at C in the plane of the object. Calculate F. [4]

7



A uniform lamina $ABCDE$ consists of a rectangle $BCDE$ and an isosceles triangle ABE joined along their common edge BE. For the triangle, $AB = AE$, $BE = a$ m and the perpendicular height is h m. For the rectangle, $BC = DE = 0.5$ m and $CD = BE = a$ m (see diagram).

(i) Show that the distance in metres of the centre of mass of the lamina from BE towards CD is

$$\frac{3 - 4h^2}{12 + 12h}.$$ [4]

The lamina is freely suspended at E and hangs in equilibrium.

(ii) Given that DE is horizontal, calculate h. [2]

(iii) Given instead that $h = 0.5$ and AE is horizontal, calculate a. [3]