Equilibrium of a Rigid Body

Question Paper 5

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Equilibrium of a Rigid Body
Sub Topic	
Booklet	Question Paper 5

Time Allowed: **57 minutes**

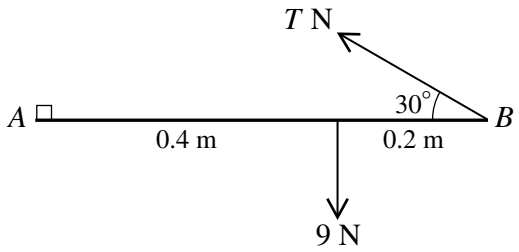
Score: **/47**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1



A non-uniform rod AB, of length 0.6 m and weight 9 N, has its centre of mass 0.4 m from A. The end A of the rod is in contact with a rough vertical wall. The rod is held in equilibrium, perpendicular to the wall, by means of a light string attached to B. The string is inclined at 30° to the horizontal. The tension in the string is T N (see diagram).

(i) Calculate T. [2]

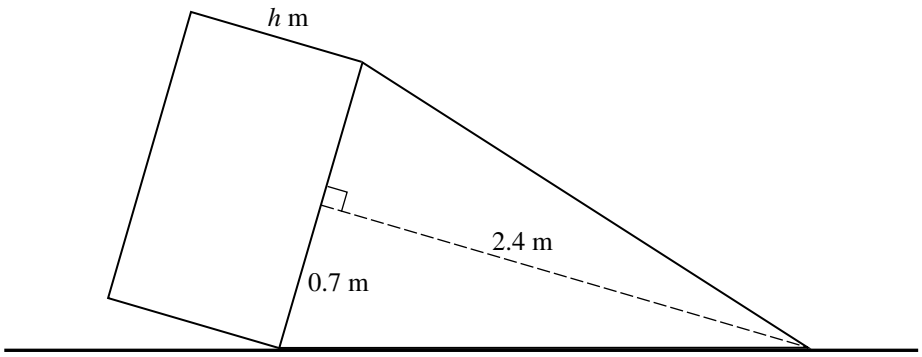
(ii) Find the least possible value of the coefficient of friction at A. [3]

2 A uniform solid cylinder has radius 0.7 m and height h m. A uniform solid cone has base radius 0.7 m and height 2.4 m. The cylinder and the cone both rest in equilibrium each with a circular face in contact with a horizontal plane. The plane is now tilted so that its inclination to the horizontal, $\theta°$, is increased gradually until the cone is about to topple.

(i) Find the value of θ at which the cone is about to topple. [2]

(ii) Given that the cylinder does not topple, find the greatest possible value of h. [2]

The plane is returned to a horizontal position, and the cone is fixed to one end of the cylinder so that the plane faces coincide. It is given that the weight of the cylinder is three times the weight of the cone. The curved surface of the cone is placed on the horizontal plane (see diagram).



(iii) Given that the solid immediately topples, find the least possible value of h. [5]

3 An object is made from two identical uniform rods AB and BC each of length 0.6 m and weight 7 N. The rods are rigidly joined to each other at B and angle $ABC = 90^\circ$.

(i) Calculate the distance of the centre of mass of the object from B. [1]

The object is freely suspended at A and a force of magnitude F N is applied to the rod BC at C. The object is in equilibrium with AB inclined at 45° to the horizontal.

(ii) (a)

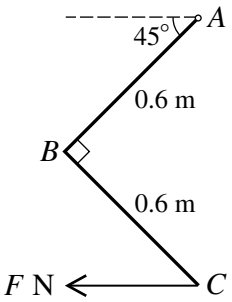


Fig. 1

Calculate F given that the force acts horizontally as shown in Fig. 1. [2]

(b)

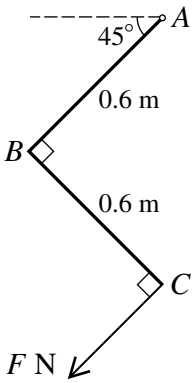


Fig. 2

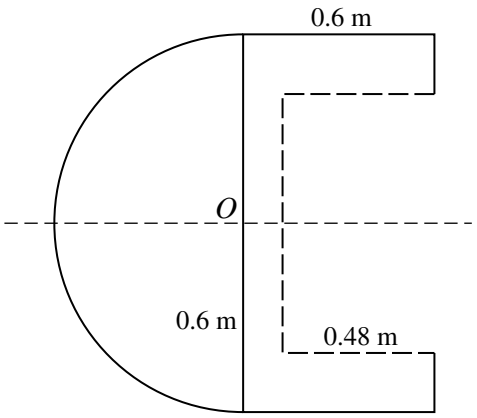
Calculate F given instead that the force acts perpendicular to the rod as shown in Fig. 2. [2]

4 A uniform solid consists of a hemisphere with centre O and radius 0.6 m joined to a cylinder of radius 0.6 m and height 0.6 m. The plane face of the hemisphere coincides with one of the plane faces of the cylinder.

(i) Calculate the distance of the centre of mass of the solid from O. [4]

[The volume of a hemisphere of radius r is $\frac{2}{3}\pi r^3$.]

(ii)



A cylindrical hole, of length 0.48 m, starting at the plane face of the solid, is made along the axis of symmetry (see diagram). The resulting solid has its centre of mass at O. Show that the area of the cross-section of the hole is $\frac{3}{16}\pi$ m^2. [4]

(iii) It is possible to increase the length of the cylindrical hole so that the solid still has its centre of mass at O. State the increase in the length of the hole. [1]

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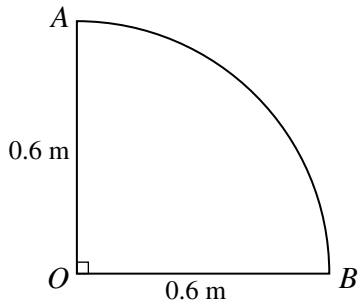

AOB is a uniform lamina in the shape of a quadrant of a circle with centre O and radius 0.6 m (see diagram).

(i) Calculate the distance of the centre of mass of the lamina from A. [3]

The lamina is freely suspended at A and hangs in equilibrium.

(ii) Find the angle between the vertical and the side AO of the lamina. [3]

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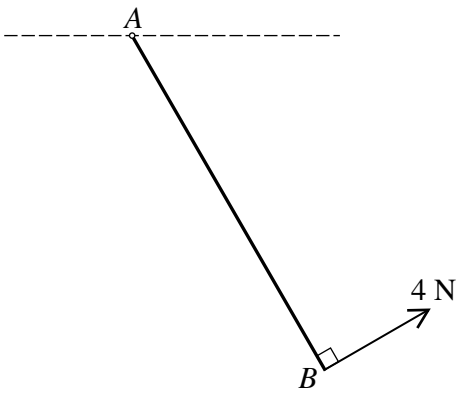


A uniform rod AB of weight 16 N is freely hinged at A to a fixed point. A force of magnitude 4 N acting perpendicular to the rod is applied at B (see diagram). Given that the rod is in equilibrium,

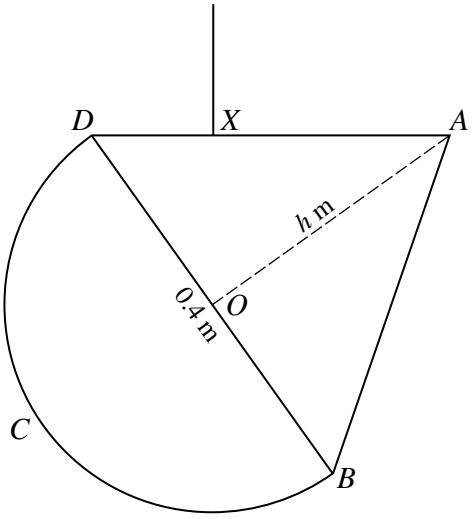
(i) calculate the angle the rod makes with the horizontal, [2]

(ii) find the magnitude and direction of the force exerted on the rod at A. [4]

7 A uniform lamina $ABCD$ consists of a semicircle BCD with centre O and diameter 0.4 m, and an isosceles triangle ABD with base BD = 0.4 m and perpendicular height h m. The centre of mass of the lamina is at O.

(i) Find the value of h. [4]

(ii)



The lamina is suspended from a vertical string attached to a point X on the side AD of the triangle (see diagram). Given the lamina is in equilibrium with AD horizontal, calculate XD. [3]