Equilibrium of a Rigid Body

Question Paper 6

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Equilibrium of a Rigid Body
Sub Topic	
Booklet	Question Paper 6

Time Allowed: **62 minutes**

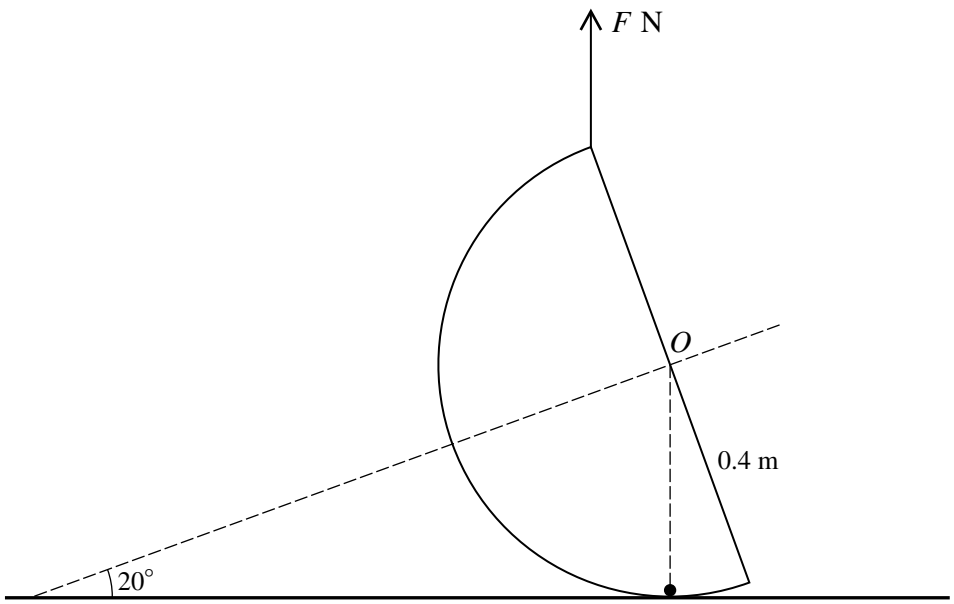
Score: **/51**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1

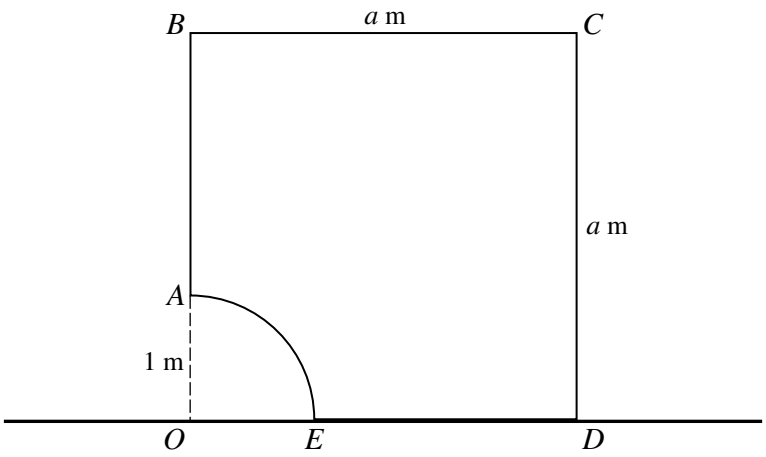


A smooth hemispherical shell, with centre O, weight 12 N and radius 0.4 m, rests on a horizontal plane. A particle of weight W N lies at rest on the inner surface of the hemisphere vertically below O. A force of magnitude F N acting vertically upwards is applied to the highest point of the hemisphere, which is in equilibrium with its axis of symmetry inclined at 20° to the horizontal (see diagram).

(i) Show, by taking moments about O, that $F = 16.48$ correct to 4 significant figures. [3]

(ii) Find the normal contact force exerted by the plane on the hemisphere in terms of W. Hence find the least possible value of W. [3]

2



$ABCDE$ is the cross-section through the centre of mass of a uniform prism resting in equilibrium with DE on a horizontal surface. The cross-section has the shape of a square $OBCD$ with sides of length a m, from which a quadrant OAE of a circle of radius 1 m has been removed (see diagram).

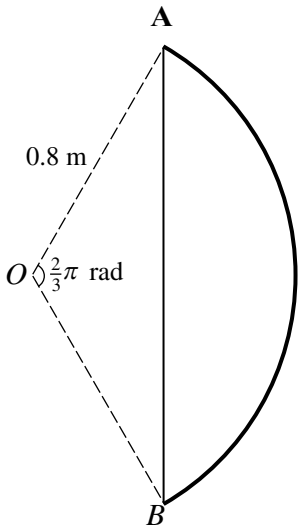
(i) Find the distance of the centre of mass of the prism from O, giving the answer in terms of a, π and $\sqrt{2}$. [5]

(ii) Hence show that

$$3a^2(2-a) < \tfrac{3}{2}\pi - 2,$$

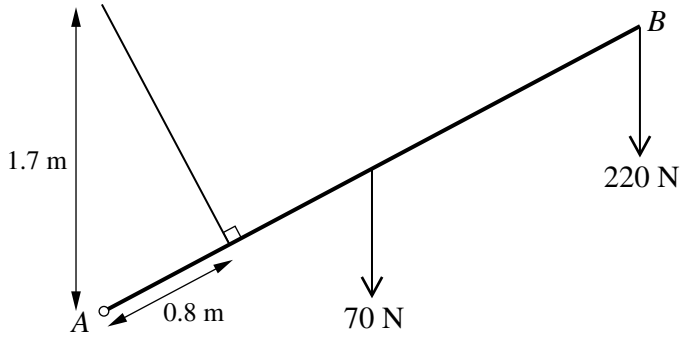
and verify that this inequality is satisfied by $a = 1.68$ but not by $a = 1.67$. [4]

3



A bow consists of a uniform curved portion AB of mass 1.4 kg, and a uniform taut string of mass m kg which joins A and B. The curved portion AB is an arc of a circle centre O and radius 0.8 m. Angle AOB is $\frac{2}{3}\pi$ radians (see diagram). The centre of mass of the bow (including the string) is 0.65 m from O. Calculate m. [6]

4

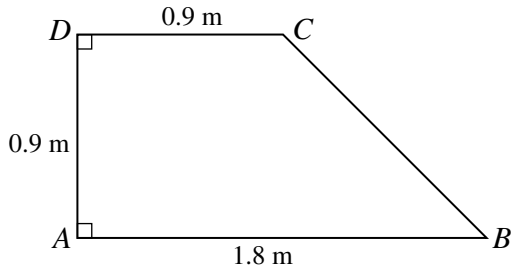


A uniform beam AB has length 2 m and weight 70 N. The beam is hinged at A to a fixed point on a vertical wall, and is held in equilibrium by a light inextensible rope. One end of the rope is attached to the wall at a point 1.7 m vertically above the hinge. The other end of the rope is attached to the beam at a point 0.8 m from A. The rope is at right angles to AB. The beam carries a load of weight 220 N at B (see diagram).

(i) Find the tension in the rope. [3]

(ii) Find the direction of the force exerted on the beam at A. [4]

5



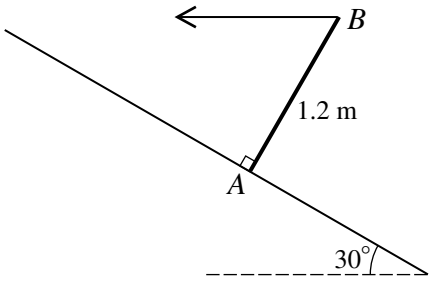
$ABCD$ is a uniform lamina with $AB = 1.8$ m, $AD = DC = 0.9$ m, and AD perpendicular to AB and DC (see diagram).

(i) Find the distance of the centre of mass of the lamina from AB and the distance from AD. [4]

The lamina is freely suspended at A and hangs in equilibrium.

(ii) Calculate the angle between AB and the vertical. [2]

6



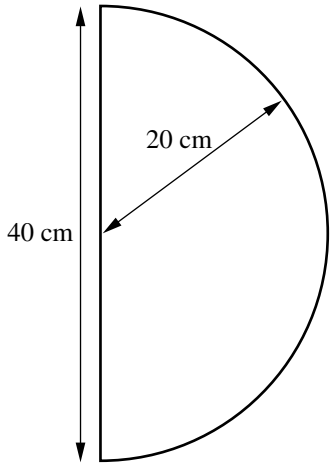
A uniform rod AB has weight 15 N and length 1.2 m. The end A of the rod is in contact with a rough plane inclined at 30° to the horizontal, and the rod is perpendicular to the plane. The rod is held in equilibrium in this position by means of a horizontal force applied at B, acting in the vertical plane containing the rod (see diagram).

(i) Show that the magnitude of the force applied at B is 4.33 N, correct to 3 significant figures. [3]

(ii) Find the magnitude of the frictional force exerted by the plane on the rod. [2]

(iii) Given that the rod is in limiting equilibrium, calculate the coefficient of friction between the rod and the plane. [3]

7



A frame consists of a uniform semicircular wire of radius 20 cm and mass 2 kg, and a uniform straight wire of length 40 cm and mass 0.9 kg. The ends of the semicircular wire are attached to the ends of the straight wire (see diagram). Find the distance of the centre of mass of the frame from the straight wire. [4]

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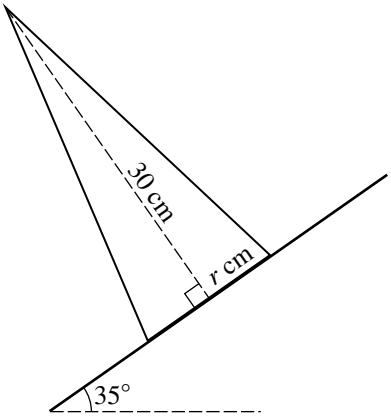

A uniform solid cone has height 30 cm and base radius r cm. The cone is placed with its axis vertical on a rough horizontal plane. The plane is slowly tilted and the cone remains in equilibrium until the angle of inclination of the plane reaches 35°, when the cone topples. The diagram shows a cross-section of the cone.

(i) Find the value of r. [3]

(ii) Show that the coefficient of friction between the cone and the plane is greater than 0.7. [2]