

Equilibrium of a Rigid Body

Question Paper 7

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Equilibrium of a Rigid Body
Sub Topic	
Booklet	Question Paper 7

Time Allowed: 60 minutes

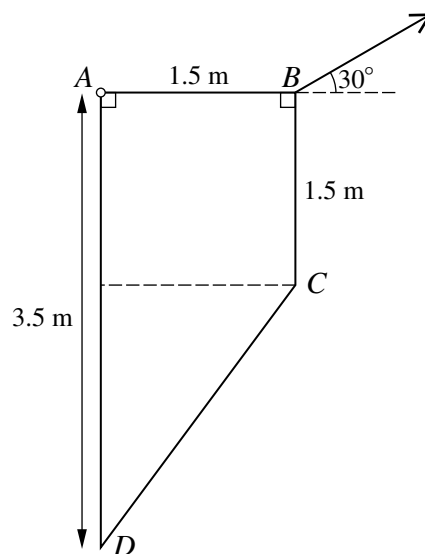
Score: /50

Percentage: /100

Grade Boundaries:

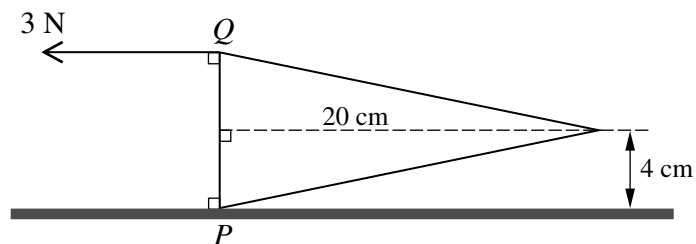
A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1



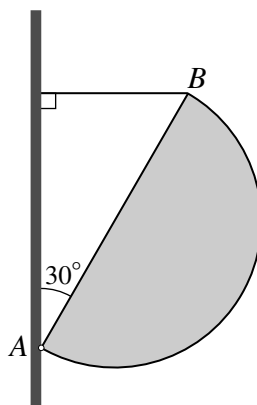
A uniform lamina of weight 15 N is in the form of a trapezium $ABCD$ with dimensions as shown in the diagram. The lamina is freely hinged at A to a fixed point. One end of a light inextensible string is attached to the lamina at B . The lamina is in equilibrium with AB horizontal; the string is taut and in the same vertical plane as the lamina, and makes an angle of 30° upwards from the horizontal (see diagram). Find the tension in the string. [5]

2



A uniform solid cone has height 20 cm and base radius 4 cm. PQ is a diameter of the base of the cone. The cone is held in equilibrium, with P in contact with a horizontal surface and PQ vertical, by a force applied at Q . This force has magnitude 3 N and acts parallel to the axis of the cone (see diagram). Calculate the mass of the cone. [4]

3



AB is the diameter of a uniform semicircular lamina which has radius 0.3 m and mass 0.4 kg. The lamina is hinged to a vertical wall at A with AB inclined at 30° to the vertical. One end of a light inextensible string is attached to the lamina at B and the other end of the string is attached to the wall vertically above A . The lamina is in equilibrium in a vertical plane perpendicular to the wall with the string horizontal (see diagram).

(i) Show that the tension in the string is 2.00 N correct to 3 significant figures. [4]

(ii) Find the magnitude and direction of the force exerted on the lamina by the hinge. [3]

4

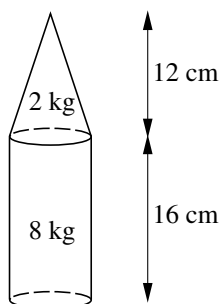


Fig. 1

A uniform solid cylinder has mass 8 kg and height 16 cm. A uniform solid cone, whose base radius is the same as the radius of the cylinder, has mass 2 kg and height 12 cm. A composite solid is formed by joining the cylinder and cone so that the base of the cone coincides with one end of the cylinder (see Fig. 1).

- (i) Show that the centre of mass of the composite solid is 10.2 cm from its base. [3]

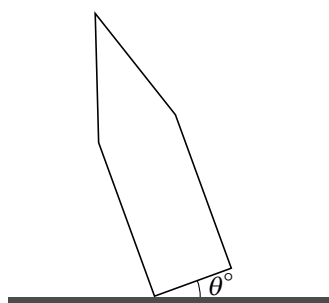
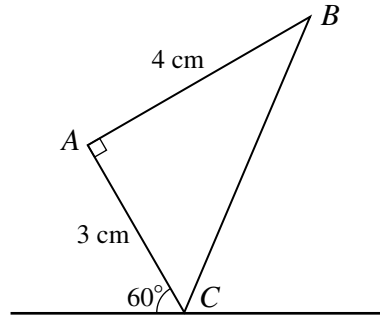


Fig. 2

The composite solid is held with a point on the circumference of its base in contact with a horizontal table. The base makes an angle θ° with the table (see Fig. 2, which shows a cross-section). When the cone is released it moves towards the equilibrium position in which its base is in contact with the table.

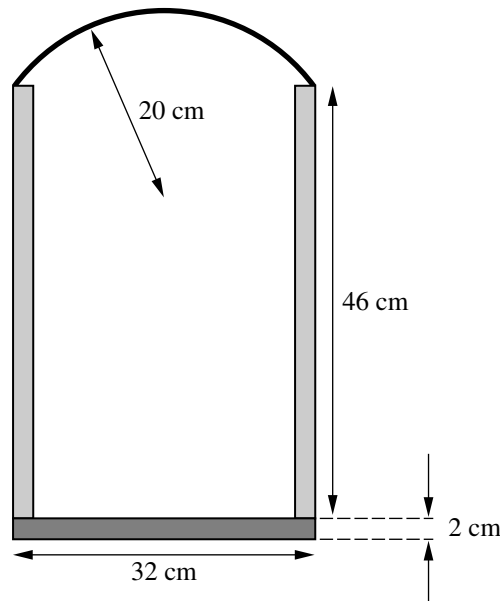
- (ii) Given that the radius of the base is 4 cm, find the greatest possible value of θ , correct to 1 decimal place. [3]

5



A uniform prism has a cross-section in the form of a triangle ABC which is right-angled at A . The sides AB and AC have lengths 4 cm and 3 cm respectively. The prism is held with the edge containing C in contact with a horizontal surface and with AC making an angle of 60° with the horizontal (see diagram). The prism is now released. Determine whether it falls on the face containing AC or the face containing BC . [4]

6



A bucket that consists of three parts stands on horizontal ground. The base is in the form of a uniform circular disc of diameter 32 cm and thickness 2 cm. The body is in the form of a uniform hollow cylinder of outer diameter 32 cm and height 46 cm. The handle is in a vertical plane, attached at opposite ends of an outer diameter at the top of the cylinder. The handle is in the form of a uniform circular arc of radius 20 cm. The diagram shows the cross-section of the bucket in the plane of the handle.

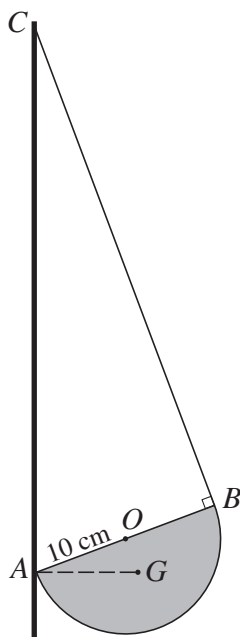
- (i) Show that the centre of mass of the handle is 53.25 cm above the ground, correct to 4 significant figures. [3]

The weights of the base, body and handle are 50 N, 100 N and 25 N respectively.

- (ii) Find the height of the centre of mass of the bucket above the ground. [2]

- 7 A uniform lamina is in the form of a sector of a circle with centre O , radius 0.2 m and angle 1.5 radians. The lamina rotates in a horizontal plane about a fixed vertical axis through O . The centre of mass of the lamina moves with speed 0.4 m s^{-1} . Show that the angular speed of the lamina is 3.30 rad s^{-1} , correct to 3 significant figures. [4]

8



AB is a diameter of a uniform solid hemisphere with centre O , radius 10 cm and weight 12 N. One end of a light inextensible string is attached to the hemisphere at B and the other end is attached to a fixed point C of a vertical wall. The hemisphere is in equilibrium with A in contact with the wall at a point vertically below C . The centre of mass G of the hemisphere is at the same horizontal level as A , and angle ABC is a right angle (see diagram). Calculate the tension in the string. [4]

9

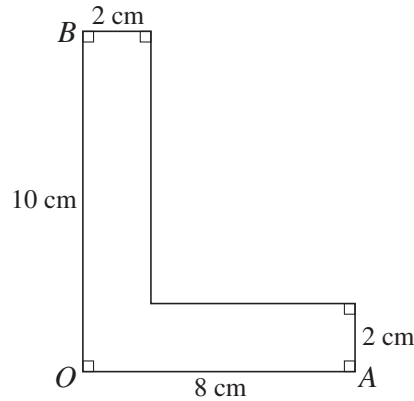


Fig. 1

A uniform solid body has a cross-section as shown in Fig. 1.

- (i) Show that the centre of mass of the body is 2.5 cm from the plane face containing OB and 3.5 cm from the plane face containing OA . [4]
- (ii) The solid is placed on a rough plane which is initially horizontal. The coefficient of friction between the solid and the plane is μ .

(a)

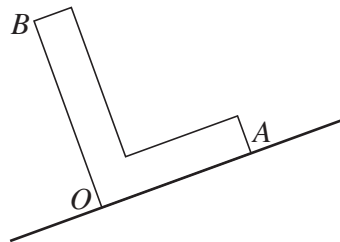


Fig. 2

The solid is placed with OA in contact with the plane, and then the plane is tilted so that OA lies along a line of greatest slope with A higher than O (see Fig. 2). When the angle of inclination is sufficiently great the solid starts to topple (without sliding). Show that $\mu > \frac{5}{7}$. [5]

(b)

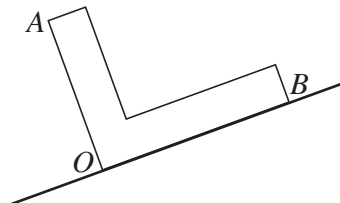


Fig. 3

Instead, the solid is placed with OB in contact with the plane, and then the plane is tilted so that OB lies along a line of greatest slope with B higher than O (see Fig. 3). When the angle of inclination is sufficiently great the solid starts to slide (without toppling). Find another inequality for μ . [2]